Future Prospects in Gravitational Waves: From Testing Fundamental Physics to Instruments beyond LIGO

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Caltech

Thesis Defense
May 05, 2025

Committee Members:

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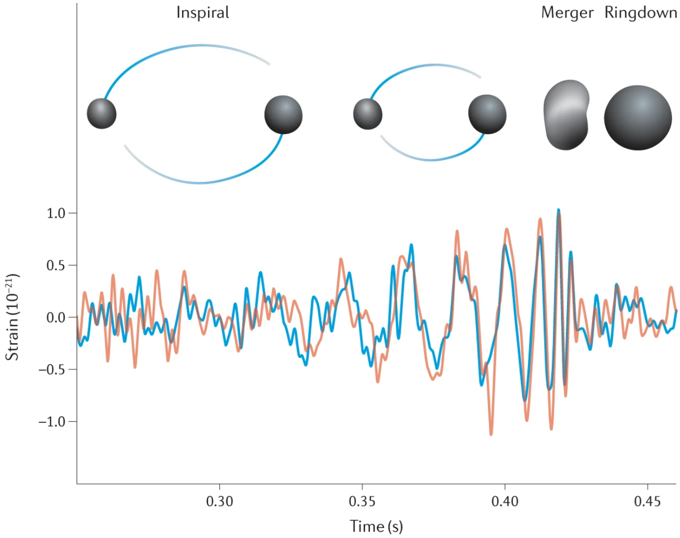
Yanbei Chen
Saul Teukolsky
Kathryn Zurek

Supervisor: Yanbei Chen

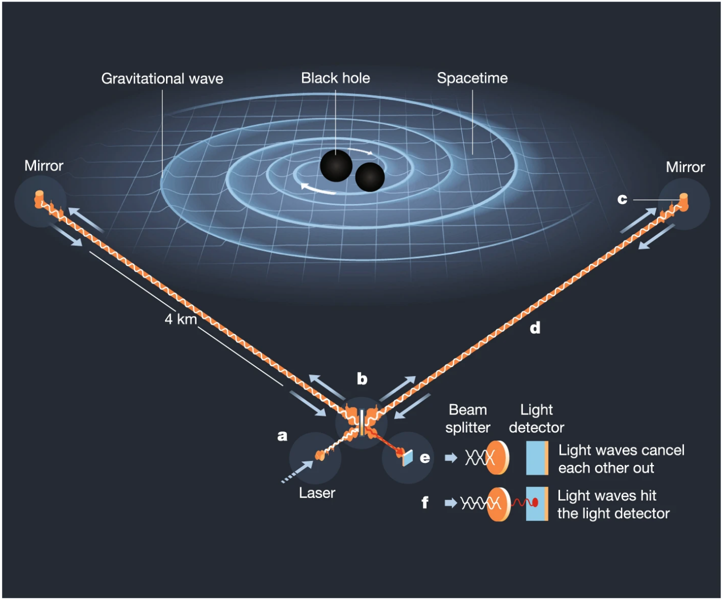
Outline

Introduction



- Image of first detection of gravitational waves in 2015.

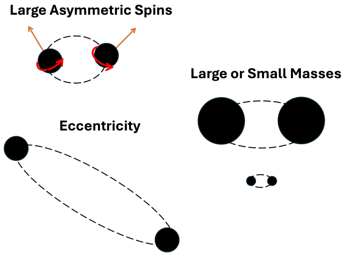
[LIGO+ 2016, Bailes+ 2021]

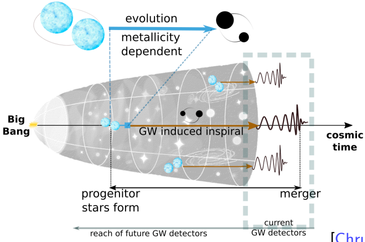

Gravitational wave
Black hole
Spacetime
Mirror
Mirror
c
4 km
d
b
a
e
Laser
Beam splitter
Light detector
Light waves cancel each other out
f
Light waves hit the light detector

[Miller+ 2019]

Current & Future Science Goals

1. Astrophysics of compact objects: formation scenarios and exotic systems
2. Nuclear physics: tidal deformability and neutron-star equation of state
3. Multimessenger astrophysics: electromagnetic counterparts and kilonova
4. Cosmology: independently measuring expansion of universe
5. Stochastic background: unresolved background of gravitational waves
6. Lensing signatures
7. Fundamental physics: testing consistency with general relativity





[Chruslinska 2022]

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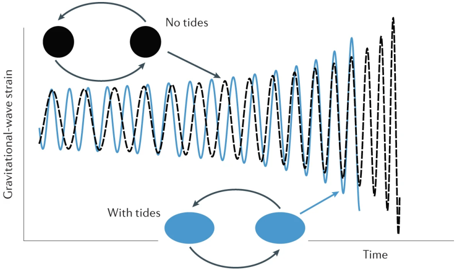

[Carson 2020]

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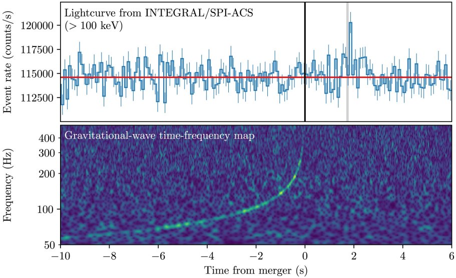

[LIGO+ 2017]

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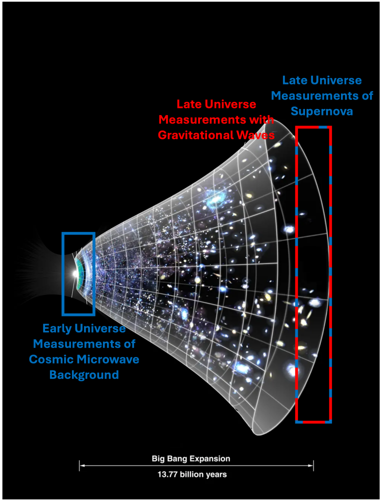

[NASA / WMAP Science Team]

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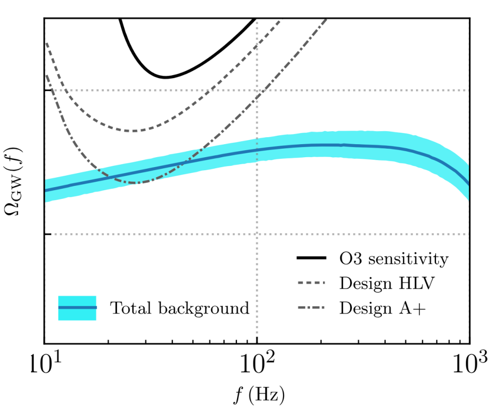

[LIGO+ 2023]

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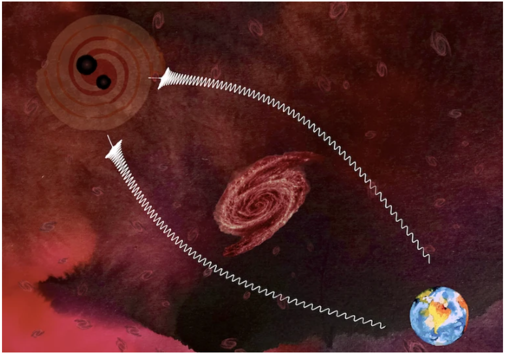

[Jana+ 2023]

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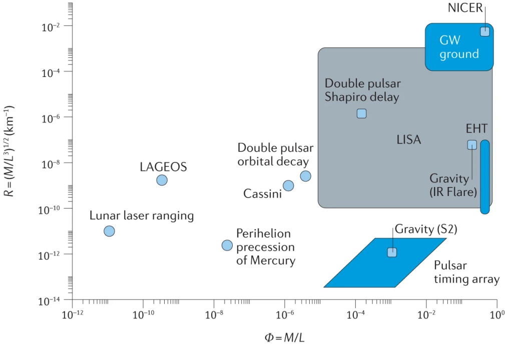
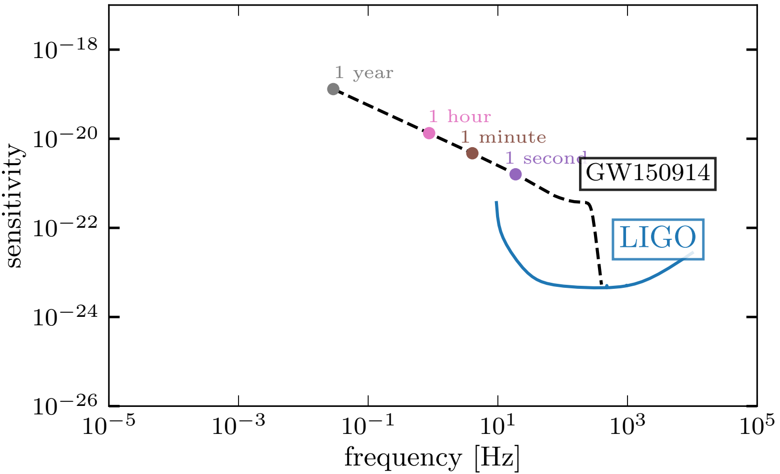


[Yunes+ 2016, Bailes+ 2021]

Current & Future Detectors



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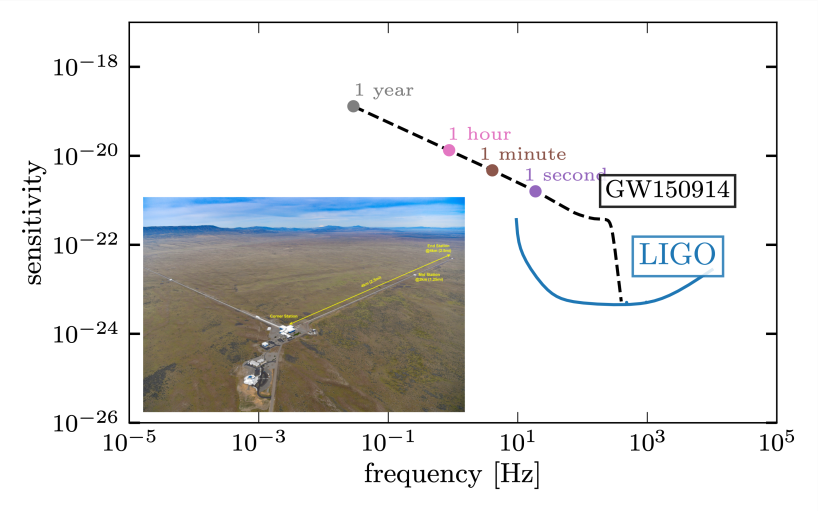

[LIGO Lab]

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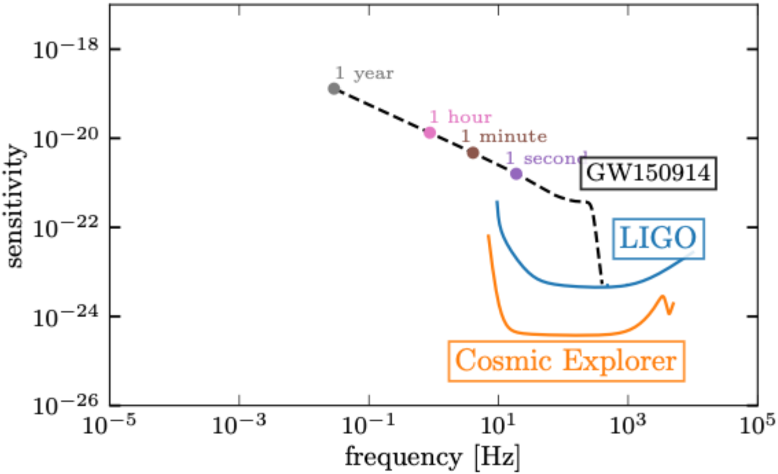

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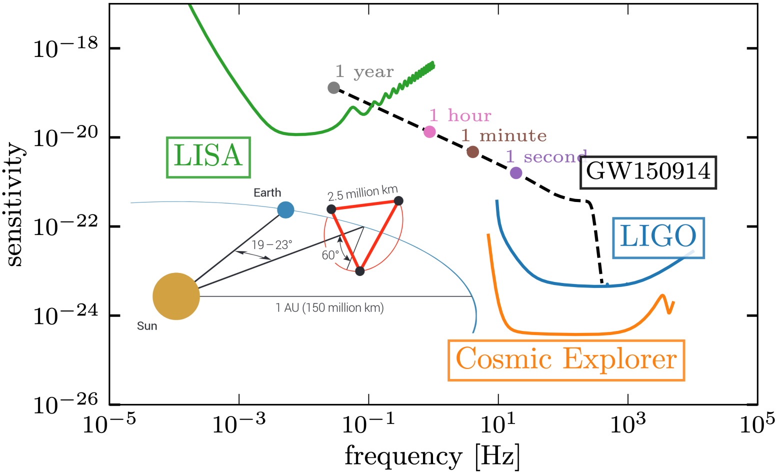

[LISA Collaboration 2017]

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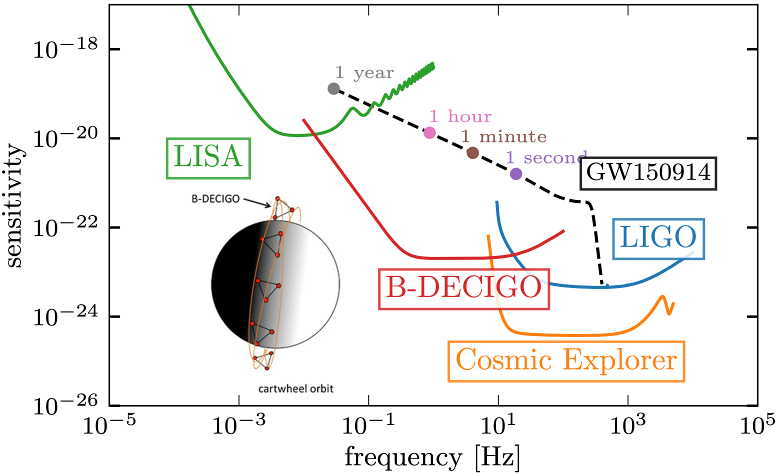

[DECIGO 2017]

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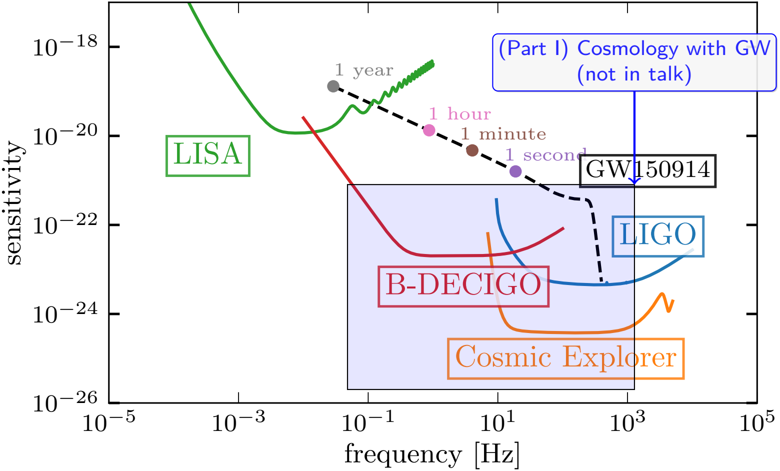

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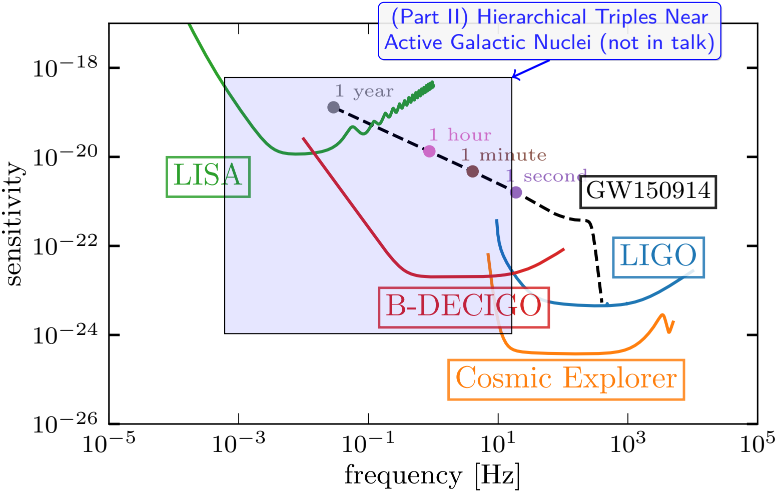

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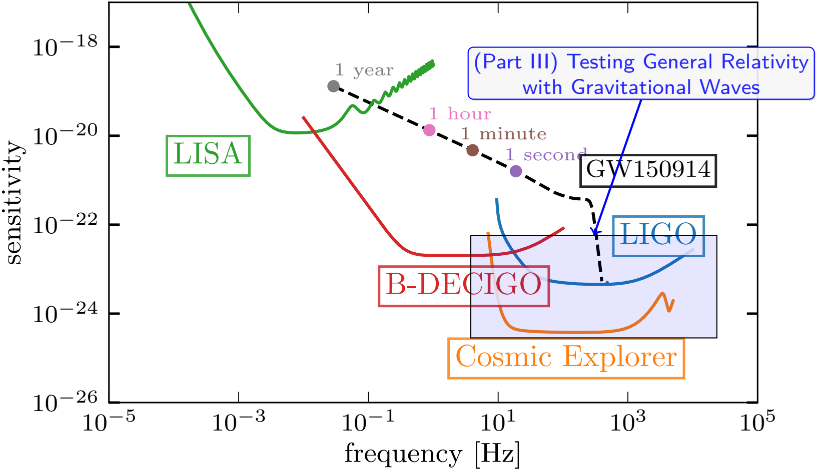

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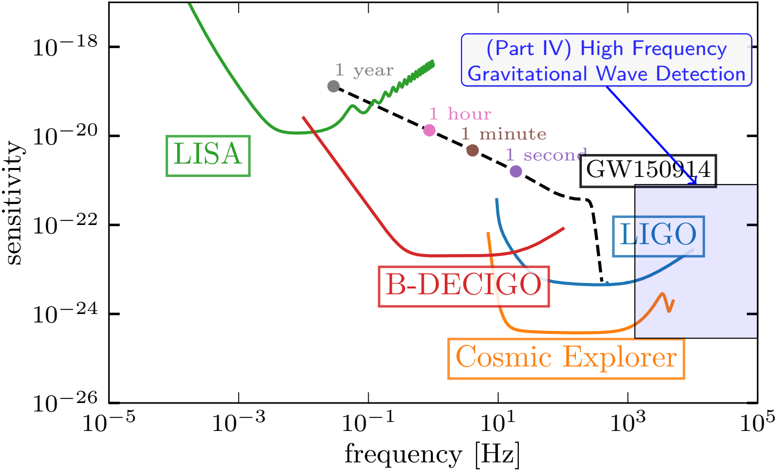
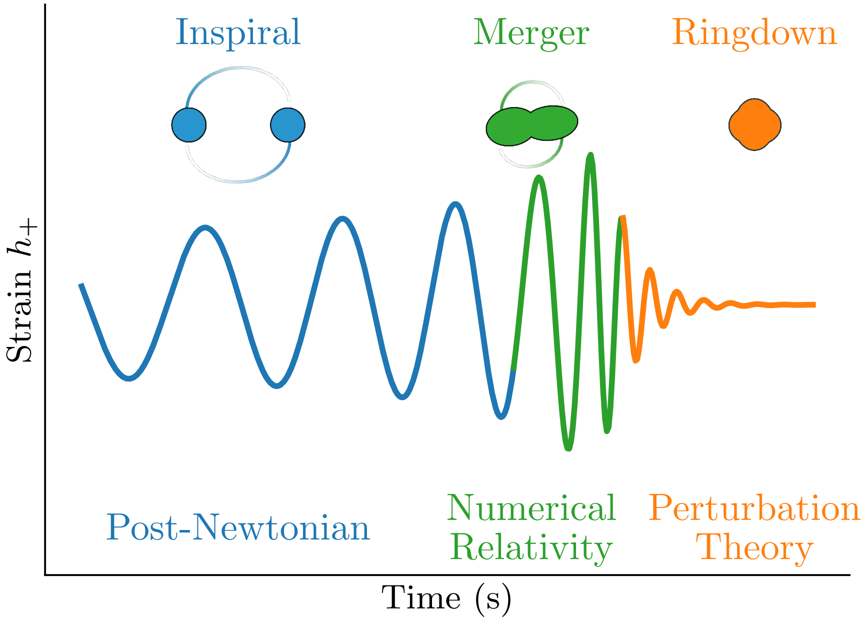
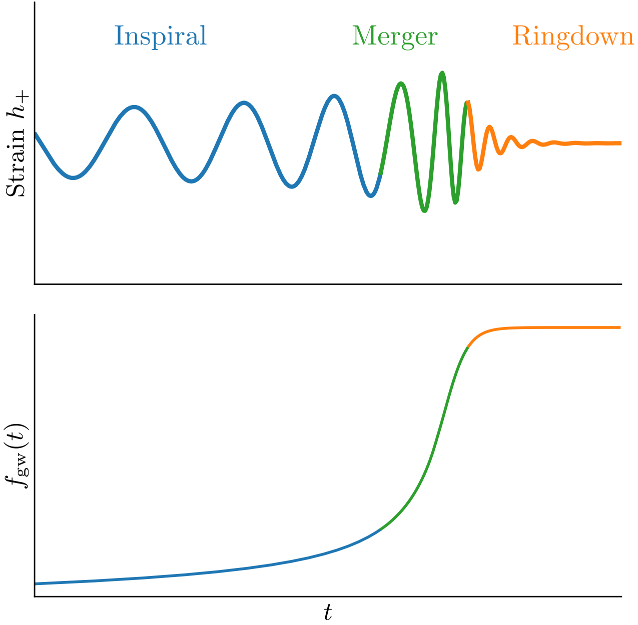



Inspiral
Merger
Ringdown
Strain h_+
Post-Newtonian
Numerical Relativity
Perturbation Theory
Time (s)


Inspiral
Merger
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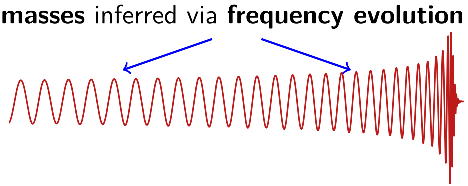
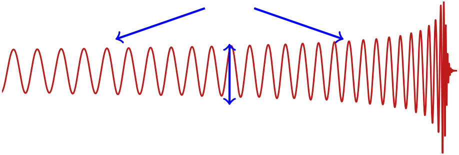
Strain h_+
$f_{\rm gw}(t)$
t

Gravitational Waves Encode Intrinsic Binary Physics



Gravitational Waves Encode Intrinsic Binary Physics

masses inferred via **frequency evolution**



distance measured via **amplitude** and **masses**

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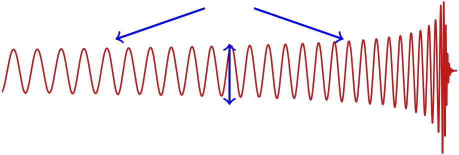

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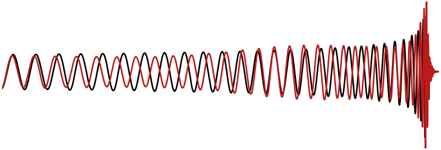

modulations of **amplitude** and **phase** encode **spins**

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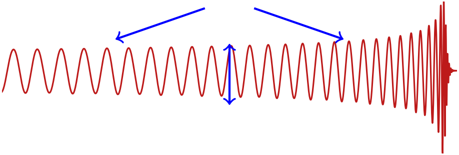

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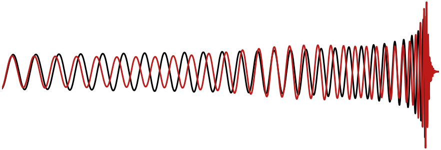

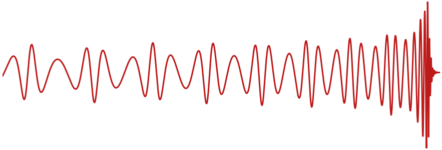

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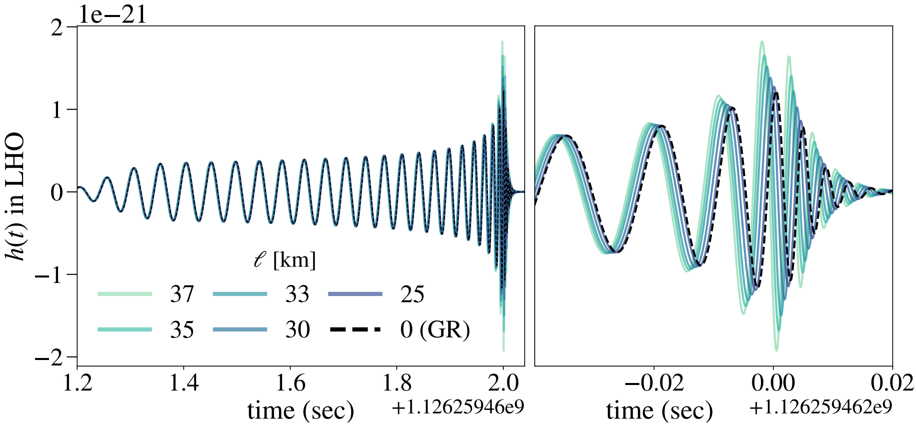
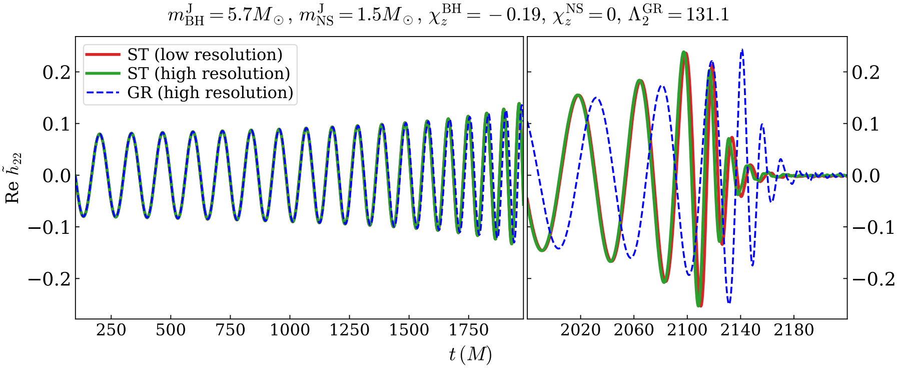
eccentricity also manifests in **amplitude** and **phase** modulations

Beyond GR Waveforms



- Beyond GR simulation of dynamical Chern-Simons [Okounkova+ 2023]

Beyond GR Waveforms



- Beyond GR simulation of scalar tensor [Ma+ 2023]

Parameterized Tests of General Relativity

- Beyond GR signals are typically characterized by dephasing

$$h_{\mathrm{bgr}}(f;\theta,\alpha) = h_{\mathrm{gr}}(f;\theta)e^{i\Delta\Psi(f;\alpha)}$$

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$(\alpha_{\rm dCS}, \alpha_{\rm ST}, \delta\varphi_k, ...)$

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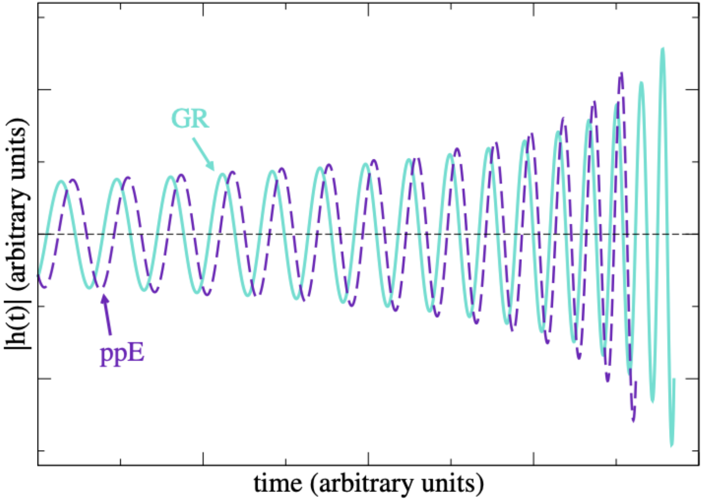
where each of these $\delta\varphi_k \leftrightarrow$ deviation of $(M/r)^k$ away from GR

PPE in Time Domain

- These ppE deviations in time domain look like [Carson+ 2020]



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- $\frac{df}{dE}$ is due to modified Kepler's third law, $\frac{dE}{dt}$ due to dissipative modifications.
- A beyond GR effect causes relative time delays $\Delta t(f)$ and then the stationary phase approximation for an adiabatic energy loss rate implies [Yunes+ 2009, Tahura+ 2019]

$$\Delta\Psi(f) = 2\pi \int df \, \Delta t(f)$$

Luminosity $\dot{E}$
GW energy loss rate
(dissipative)

Energy $E(f)$
Kepler's third law
(conservative)

$\frac{df}{dt} \propto (\pi M f)^{11/3} (1 + \epsilon v(f))$
frequency chirp rate

$$t(f) = t_{\mathrm{gr}}(f) + \Delta t(f)$$

$$\tilde{h}(f) = \tilde{h}_{\mathrm{gr}}(f) e^{i \Delta \Psi(f)}$$

Part A : Searching for Nonviolent Nonlocality in the Gravitational Waves

Possibilities for the Black Hole Unitarity Crisis

It is proposed that the following three statements cannot be true simultaneously [Almheiri+ 2012]

1. Hawking radiation is a pure state.
2. Infalling observer feels nothing unusual at the horizon.
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3. Hawking radiation comes from near the horizon.
    - Horizon structure of a black hole is changed, e.g. nonviolent nonlocality.

Nonviolent Nonlocality

- Nonviolent nonlocality is a proposal by Steve Giddings that posits that the information is transferred via soft modes in the black hole atmosphere [Giddings 2012, Giddings+ 2016].
- These metric fluctuations have an extent to $\sim r_S$ in contrast to the fluctuations in a firewall with extent $l_p \ll r_S$
- We have background metric, and the quantum fluctuations modify it

$$g_{\mu\nu} = g_{\mu\nu}^{\text{schw}} + n_{\mu\nu}$$

Modifications to the Metric due to Quantum Structure

- One can construct the most general metric fluctuations [Regge+ 1957], but the dominant one in ingoing Eddington-Finkelstein coordinates is [Giddings+ 2016]

$$n_{vv} = \sum_{\ell m} n_{vv}^{\ell m}(v, r) Y_{\ell m}(\phi, \theta)$$

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$$n_{vv} = \sum_{\ell m} n_{vv}^{\ell m}(v, r) Y_{\ell m}(\phi, \theta)$$

- We parameterize the random noise fluctuations as

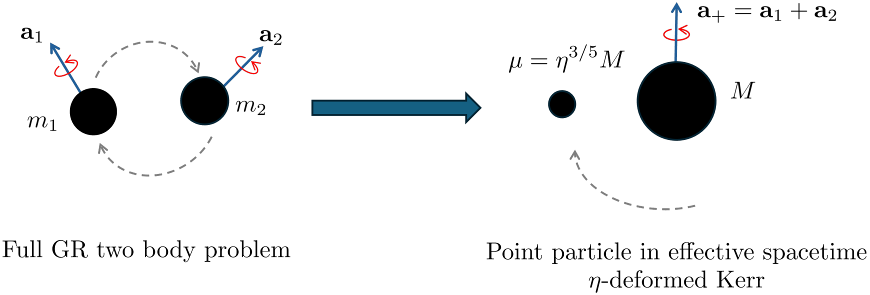
$$n_{vv}^{\ell m}(v, r) = A \exp\left[-\frac{1}{2r_S^2}(r - r_S)^2\right] n(t)$$

$$n(t) = \text{Colored gaussian noise; } \langle |n(t)| \rangle = 1$$

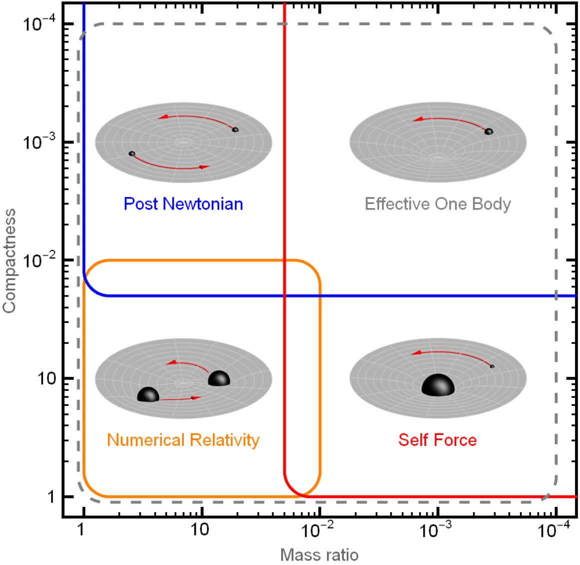
$$S_n(f) \propto \frac{1}{2f_Q} \exp\left[-|f|/f_Q\right]$$

Effective One-Body

- The effective one-body formalism is an analytical approach to finding the motion and gravitational waves in GR [Buonanno+ 1998, 2000, Damour 2001].
- It resums independent information from (a) post-Newtonian theory and (b) black hole perturbation theory



Effective One-Body



Effective One-Body Details

- The effective one-body defines an effective metric

$$ds^2_{\text{eff}} = -A(r)dt^2 + \frac{D(r)}{A(r)}dr^2 + r^2 d\Omega^2$$

where if $\eta \to 0$ it approaches the Schwarzschild metric in the nonspinning case.

- One can find that solving the mass shell condition $p_\mu p_\nu g^{\mu\nu} = -1$ yields an effective Hamiltonian

$$\hat{H}_{\text{eff}} = \sqrt{A(r)\left(1 + \frac{p_\phi^2}{r^2} + \frac{A}{D}p_r^2\right)}$$

- The particle follows the trajectory of the real Hamiltonian

$$\hat{H}_{\text{real}} = \frac{1}{\eta}\sqrt{1 + 2\eta\left(\hat{H}_{\text{eff}} - 1\right) + \mathcal{O}(\vec{P}^2_{\text{COM}})}$$

where the center of mass motion is ignored.

Perturbations to Hamilton's Equations

- Nonviolent nonlocality adds perturbations to black holes which changes the mass shell condition

$$p_\mu p_\nu \left(g_0^{\mu\nu} + n^{\mu\nu}\right) = -1$$

This causes the real Hamiltonian to be modified

$$\hat{H}_{\text{real}} = \hat{H}^0_{\text{real}} + n^{\ell m}_{vv} \, \Delta \hat{H}^{\text{real}}_{\ell m} \, .$$

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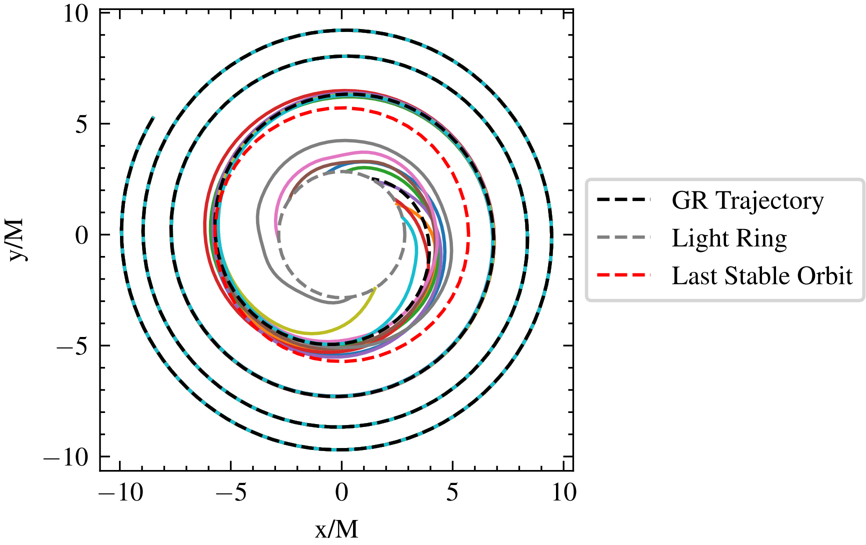
- In the end, we integrate Hamilton's equations which have the form

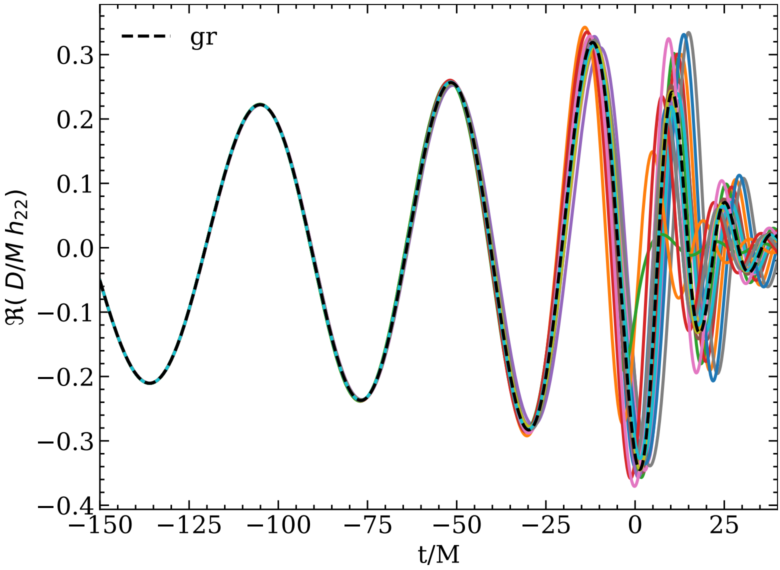
$$\frac{dq_i}{dt} = \frac{\partial \hat{H}_{\rm real}}{\partial p_i}\,,$$
$$\frac{dp_i}{dt} = -\frac{\partial \hat{H}_{\rm real}}{\partial q_i} + \mathcal{F}_i^{\rm rad}\,,$$

where $\mathcal{F}_i^{\rm rad}$ are the nonconservative forces from radiation reaction.

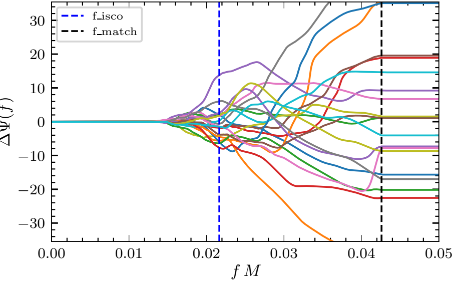
Inspiral with Quantum Fluctuations



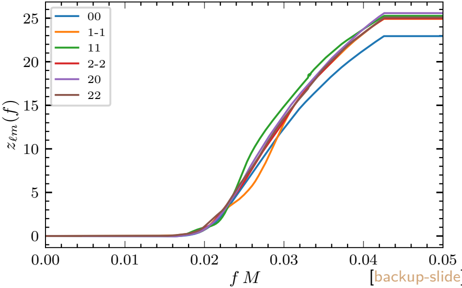
Change in Gravitational Wave Strain



Random Dephasing in Frequency Domain



PCA



- Principal component analysis yields deviation that looks like $\Delta\Psi(f) = \zeta z(f)$
- $\zeta \sim \mathcal{N}(0, A_{\rm t})$ random variable for each event

[backup-slide]

Fisher Analysis

- We inject $\theta = (\zeta, \mathcal{M}_c, q, D_l, \iota, \psi, \alpha, \delta)$ from realistic astrophysical populations and compute the estimated variance with the Fisher matrix

$$\Gamma_{ij} = \left(\partial_{\theta_i} h | \partial_{\theta_j} h\right)$$

  where $(.|.)$ is the standard noise weighted inner product.

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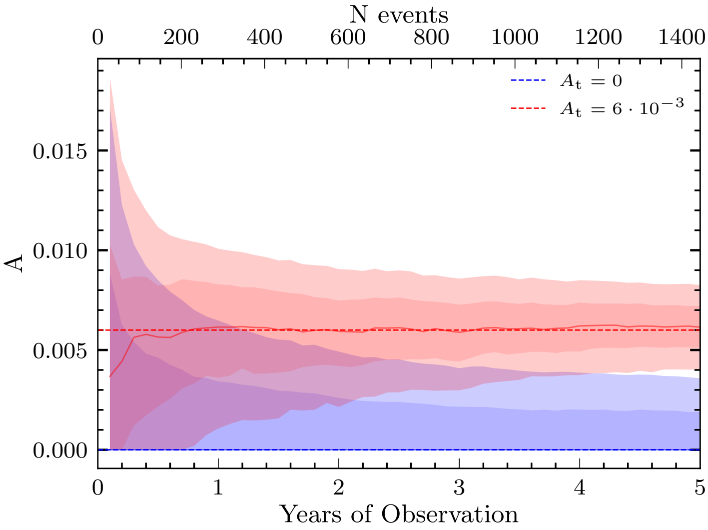
where $(.|.)$ is the standard noise weighted inner product.

- From this we calculate the marginalized likelihood $p(d_a|\zeta)$ for each event a. The likelihood for the hyper parameters is

$$p(d|\mu, \sigma) = \int d\zeta p(d|\zeta) p(\zeta|\mu, \sigma)$$

- From this, we compute the posterior on $A \equiv \sigma$ by combining many events d_a.

Constraints on Nonviolent Nonlocality with LIGO



[backup-slide]

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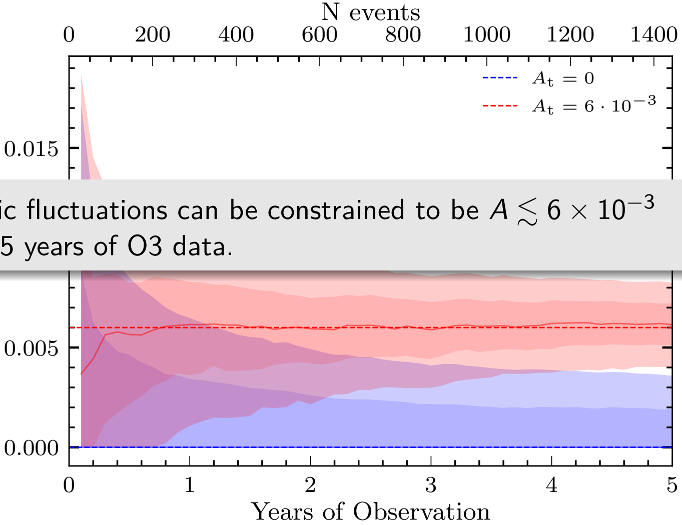

Metric fluctuations can be constrained to be $A \lesssim 6 \times 10^{-3}$ with 5 years of O3 data.

[backup-slide]

Part B : Geometric Description of Tests of GR

Testing GR with Incorrect Signal Models

- Gravitational wave data is a combination of signal and noise

$$d = s + n$$
$$s = h_{\mathrm{sig}}(\theta_{\mathrm{tr}}, \alpha_{\mathrm{tr}}) = h_{\mathrm{gr}}(f; \theta_{\mathrm{tr}}) e^{i\Delta\Psi_{\mathrm{bgr}}}$$

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1. How **degenerate with GR** parameters is $\Delta\Psi_{\rm bgr}$?
2. How **accurately** ppE $\Delta\Psi_k$ captures the true deviation $\Delta\Psi_{\rm bgr}$?

Residual Signal

- Doing PE on $h_{\rm sig}(\theta_{\rm tr}, \alpha_{\rm tr})$ with a GR waveform $h_{\rm gr}(\theta)$, the residual signal

$$\begin{aligned}\Delta h &= h_{\rm sig}(\theta_{\rm tr}, \alpha_{\rm tr}) - h_{\rm gr}(\theta_{\rm tr}) \\ &\approx i\alpha_{\rm tr}\psi_\alpha h_{\rm gr}(\theta_{\rm tr})\end{aligned}$$

- GR parameters are biased due to systematic error [Cutler+ 2007]

$$\theta_{\rm meas} \to \theta_{\rm tr} + \Delta\theta_{\rm bias}$$

- The residual signal is perpendicular part of the waveform

$$\Delta h_{\perp{\rm gr}} = \Delta h - \Delta\theta^i_{\rm bias}\partial_i h_{\rm gr}$$

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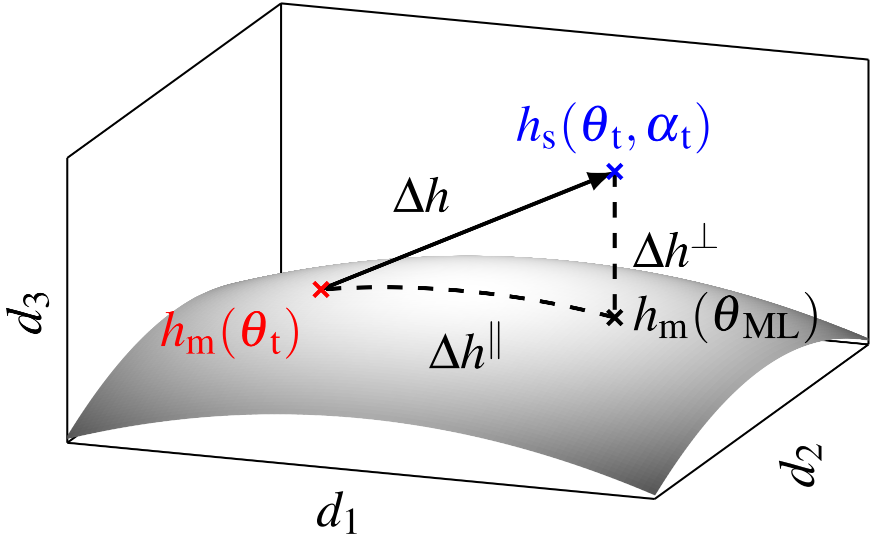
$$\Delta h_{\perp \rm gr} = \Delta h - \Delta\theta^i_{\rm bias}\partial_i h_{\rm gr}$$

- For example if you don't model spins and just try to measure $\mathcal{M}_c$ and q, the answers will be biased.


$h_s(\theta_t, \alpha_t)$
Δh
$\Delta h^{\perp}$
$h_m(\theta_{ML})$
$h_m(\theta_t)$
$\Delta h^{\parallel}$
d_3
d_1
d_2

Perpendicular SNR and Bayes Factors

- Given an injected signal $h_{\rm sig}(\theta_{\rm tr}, \alpha_{\rm tr})$, we compute the evidence for both GR and bGR [$p(\mathrm{GR}|d)$ and $p(\mathrm{bGR}|d)$]
- The Bayes factor compares the evidence for a beyond GR theory in the data

$$\mathcal{O}_{\rm GR}^{\rm bGR} \equiv \frac{p(\mathrm{bGR}|d)}{p(\mathrm{GR}|d)}$$

- The Bayes factor behaves like [Vallisneri 2009, 2013]

$$\log \mathcal{O}_{\rm GR}^{\rm bGR}|_{s_{\rm bGR}} \sim \frac{1}{2}\rho_\perp^2 + \rho_\perp x + \frac{1}{2}x^2 \qquad \text{(bGR injection)}$$

$$\log \mathcal{O}_{\rm GR}^{\rm bGR}|_{s_{\rm GR}} \sim \frac{1}{2}x^2 \qquad \text{(GR injection)}$$

where $\rho_\perp = \|s_{\rm bGR}^\perp\|$ and x is random unit normal variable.

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- How **accurately** does ppE $\Delta\Psi_k$ captures the true deviation $\Delta\Psi_{\rm bgr}$?

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where the captured SNR is

$$\rho_{\perp}^{\rm ppE} = {\rm FF}(\Delta h_{\rm bGR}^{\perp}, \Delta h_{\rm ppE}^{\perp})\,\rho_{\perp}$$

- The *fitting factor* describes how much of the bGR signal is captured by the ppE model

$${\rm FF}(\Delta h_{\rm bGR}^{\perp}, \Delta h_{\rm ppE}^{\perp}) = \frac{\left(\Delta h_{\rm bGR}^{\perp}|\Delta h_{\rm ppE}^{\perp}\right)}{\|\Delta h_{\rm bGR}^{\perp}\|\|\Delta h_{\rm ppE}^{\perp}\|}$$

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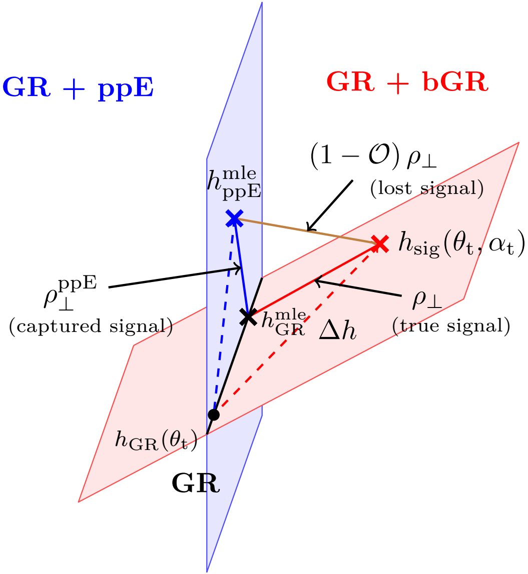
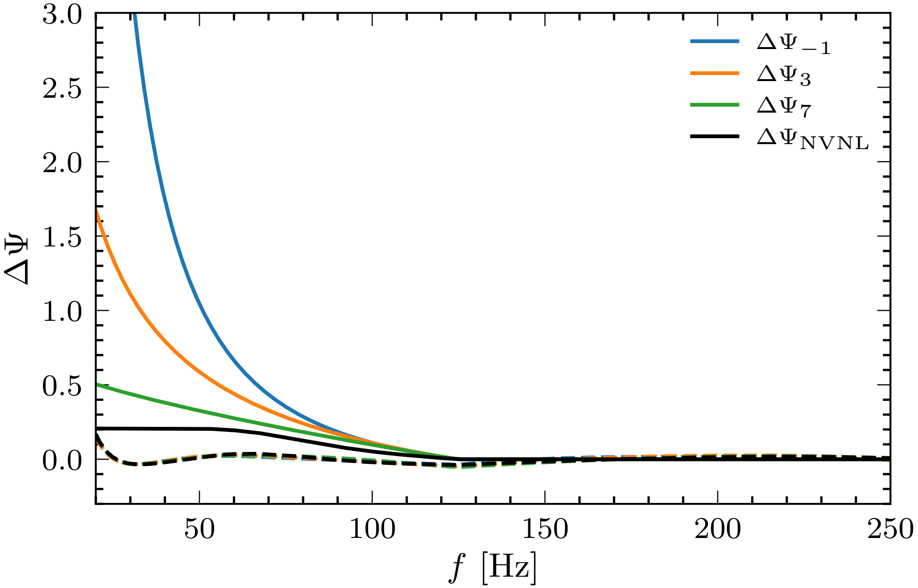
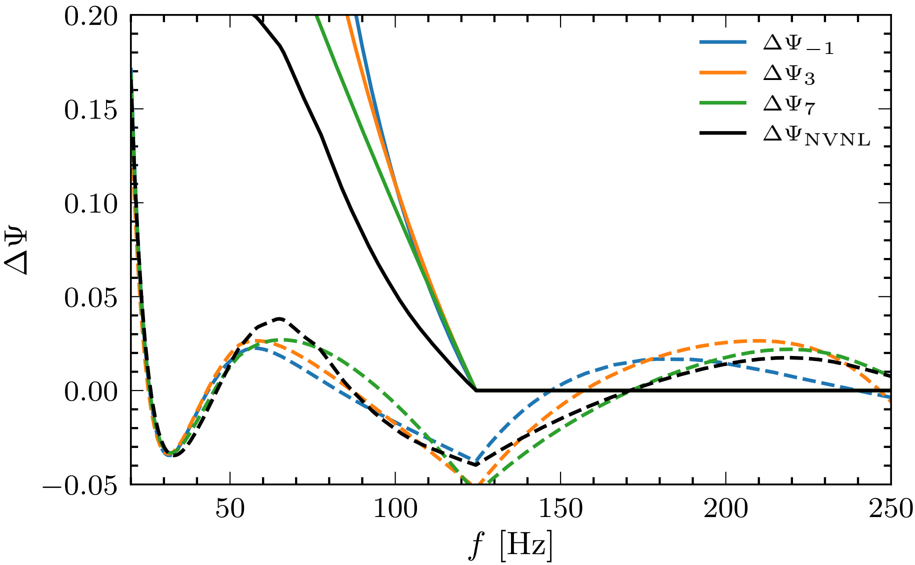
- We have found that that the sensitivity loss is very small $(1 - {\rm FF} \ll 1)$ for some non-PN theories [Seymour+ 2024].
  - $\Delta\Psi_{\rm NVNL} \sim e^{-f^{-1}}$ has essential singularity at $f = 0 \implies$ no ppE power

Visualization of Test




3.0
2.5
2.0
1.5
1.0
0.5
0.0
$\Delta\Psi$
50
100
150
200
250
f [Hz]
$\Delta\Psi_{-1}$
$\Delta\Psi_3$
$\Delta\Psi_7$
$\Delta\Psi_{\rm NVNL}$


0.20
0.15
0.10
0.05
0.00
−0.05
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50
100
150
200
250
f [Hz]
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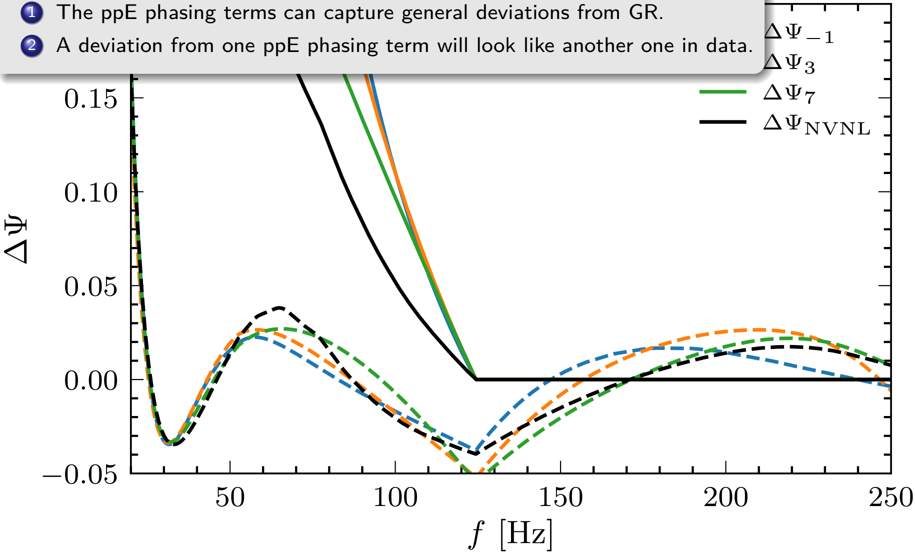
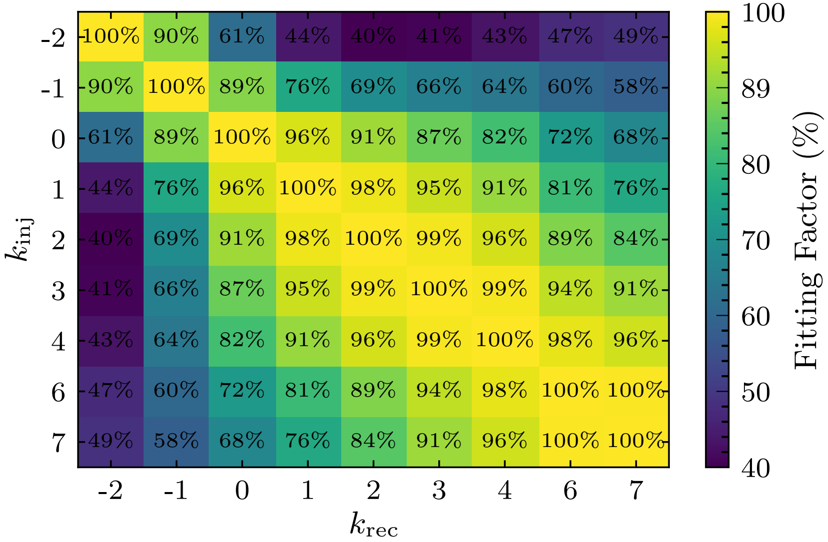
$\Delta\Psi_{\mathrm{NVNL}}$

Monotonic phase deviations have some universal features in how they deviate fom GR.

1. The ppE phasing terms can capture general deviations from GR.
2. A deviation from one ppE phasing term will look like another one in data.



Degeneracy of Multiparameter ppE



Singular Value Decomposition Approach

- Since the ppE parameter tests are degenerate, we need to identify common modes of the waveform
- We generalize the singular value decomposition [Pai+ 2013] to identify nondegenerate deviations from GR

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$$C(\Delta h_\alpha) = \sum_a \|\Delta h_a - \mathcal{P}_n \Delta h_a\|^2$$

where the projection $\mathcal{P}_n$ projects to an orthonormal SVD basis

$$\mathcal{P}_n \Delta h_a = \sum_\alpha (\Delta h_a | \Delta h_\alpha)\, \Delta h_\alpha$$

$$(\Delta h_\alpha | \Delta h_\beta) = \delta_{\alpha\beta}$$

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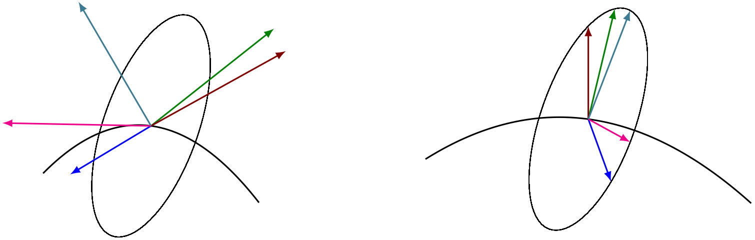
$$(\Delta h_\alpha | \Delta h_\beta) = \delta_{\alpha\beta}$$

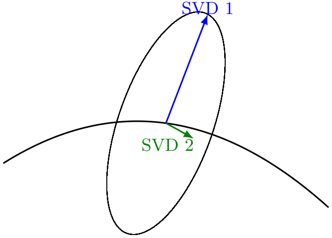
- Goal is to find $n_{\mathrm{svd}} \ll n_{\mathrm{ppE}}$ but still fit signal well

Visualization of Multiparameter SVD



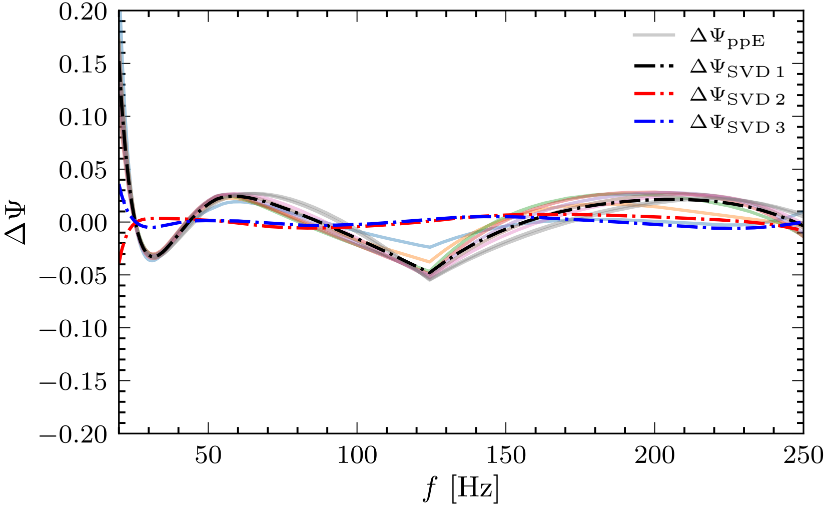
Step 1: compute $\Delta h_{\rm ppE}$

Step 2: $\Delta h^{\perp}_{\rm ppE}$ computed by projecting terms parallel to $\partial_{\theta^i} h_{\rm gr}$

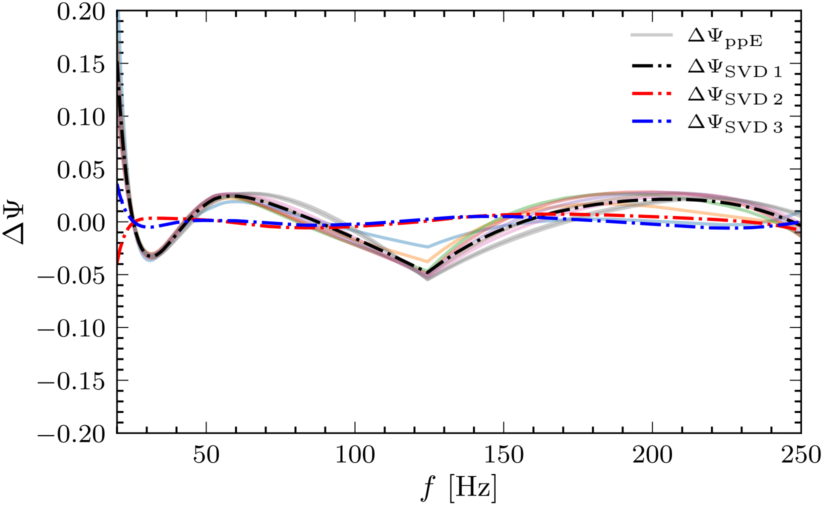


Step 3: Compute $\Delta h^{\perp}_{\rm SVD}$ with the singular value decomposition

Multiparameter SVD Example for GW150914

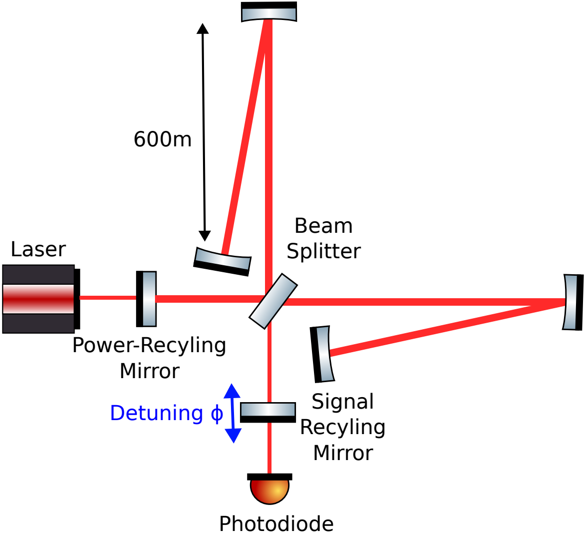


These SVD waveform modes are orthogonal so that $(i\Delta\Psi_{\mathrm{SVD}\,a} h_{\mathrm{gr}} | i\Delta\Psi_{\mathrm{SVD}\,b} h_{\mathrm{gr}}) = s_a^2 \delta_{ab}$



Part C : High Frequency Gravitational Wave Detection

GEO600 Optical Layout



How Detectors Measure Gravitational Waves

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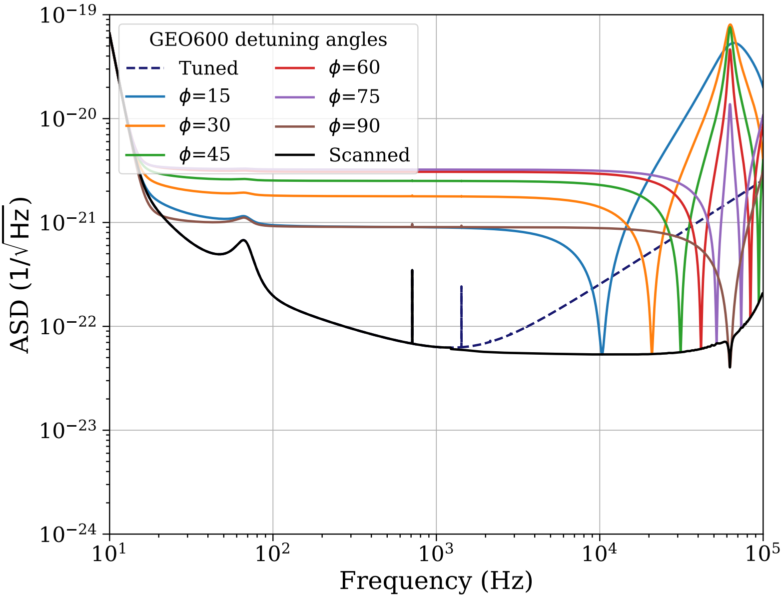
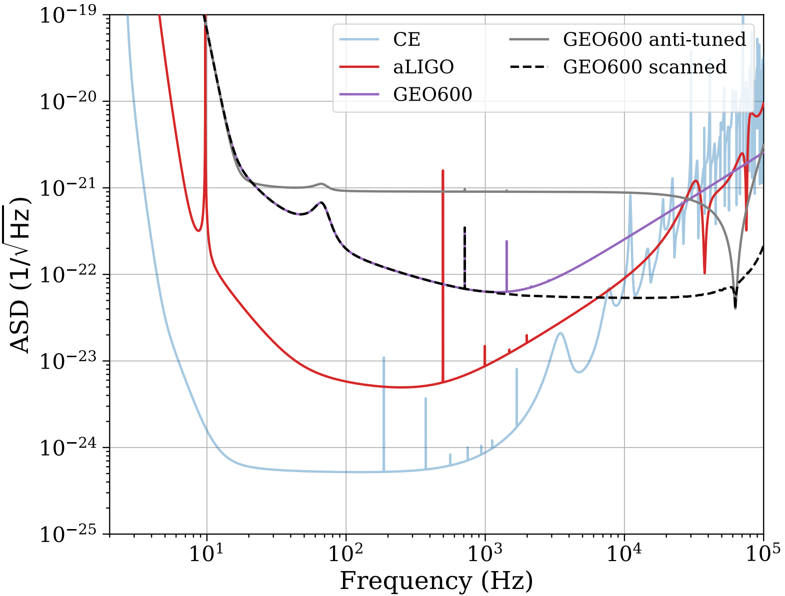
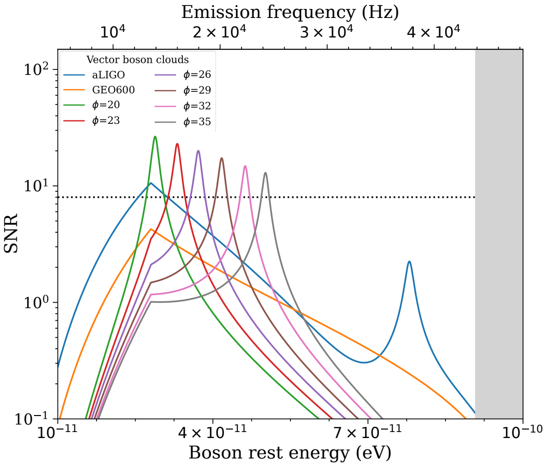
- Can we modify the **optimal frequency** that we are sensitive at by detuning the **location** of the signal recycling mirror?
- $T_h(f; \phi_{\rm srm})$ can be adjusted to search for gravitational waves at high frequencies [Meers 1988, Mizuno+ 1993].



[backup-slide]



[backup-slide]



- GEO600 detector improves the detection prospects on boson clouds ($M = 0.3\,M_\odot$, $\chi = 0.7$ using model from [Isi+ 2018])
- Quasicircular sub-solar mass mergers $10^{-3} M_\odot \lesssim M \lesssim 10^{-1}\,M_\odot$ have no improvement in sensitivity.

[backup-slide]

Conclusions

1. Studied the prospects for detecting nonviolent nonlocality in LIGO
    - Modeled the waveform in the effective-one-body formalism
    - Showed that stochastic deviations to phase are expected
    - Stacked together many events to estimate constraints in LIGO
2. Identified geometrical meaning of tests of GR
    - Explained degeneracies and significance by geometrical framework
    - Characterized systematic error of using parameterized models
    - Used singular value decomposition to identify common modes
3. Modeled the effects of high frequency sensitivity in GEO600 by modulating the signal recycling cavity location.

Thanks

- Yanbei Chen for supervising this thesis
- Previous research mentors: Hang Yu, Kent Yagi, Marie Kasprzack, Arnaud Pele, Adam Mullavey, and Klebert Feitosa.
- Committee members: Katerina Chatziioannou, Saul Teukolsky, and Kathryn Zurek
- JoAnn Boyd and other TAPIR administrative staff
- NSF GRFP and other science funding bodies.

Appendix/Backup Slides

Frequency Domain Dephasing Model

- From the figure before, we saw that the random metric fluctuations produce dephasing which has a lot of structure.
- We use a principal component analysis to model the dephasing in a simple manner.

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$$\Sigma(f, f') = \langle (\Delta\Psi(f) - \mu(f)) \left(\Delta\Psi(f') - \mu(f')\right) \rangle$$

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- The principal component analysis finds optimal eigenvectors

$$\Sigma(f, f') \approx A^2 \, z(f) z(f') \,,$$

Hierarchical Tests of GR

- What we have shown is that nonviolent nonlocality predicts stochastic deviations to the phase $\Delta\Psi(f) = \zeta z(f)$.

$$\zeta \sim \mathcal{N}(0, A)$$

- This is of the same form ot the hierarchical tests of GR [Isi+ 2019] which are published in the LIGO papers [LIGO+ 2021].

$$\delta\phi_k \sim \mathcal{N}(\mu_k, \sigma_k)$$

where these are the deformation coefficients.

Fisher Analysis

- We inject $\theta = (\zeta, \mathcal{M}_c, q, D_l, \iota, \psi, \alpha, \delta)$ from realistic astrophysical populations and compute the estimated variance with the Fisher matrix

$$\Gamma_{ij} = \left(\partial_{\theta_i} h | \partial_{\theta_j} h\right)$$

where $(.|.)$ is the standard noise weighted inner product.

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where $(.|.)$ is the standard noise weighted inner product.

- From this we calculate the marginalized likelihood $p(d_a|\zeta)$ for each event a. The likelihood for the hyper parameters is

$$p(d|\mu, \sigma) = \int d\zeta p(d|\zeta) p(\zeta|\mu, \sigma)$$

- From this, we compute the posterior on the hyper parameters

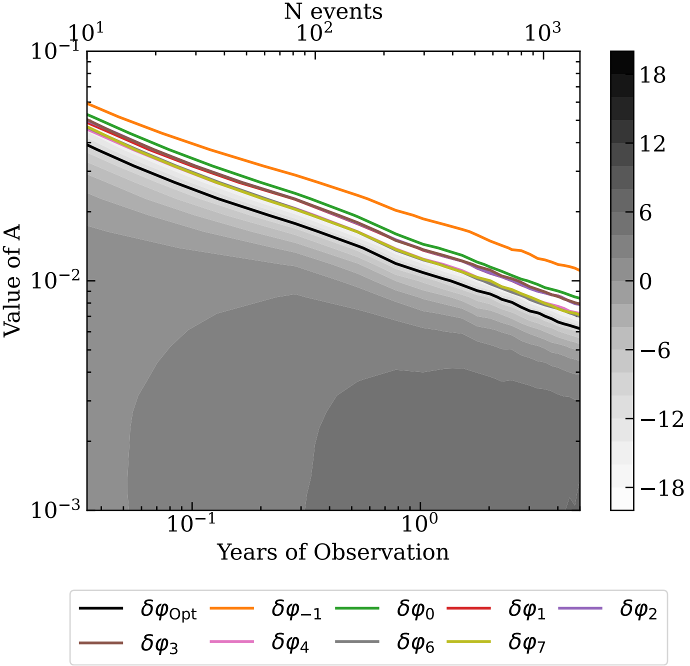
$$p(\{d\}|\mu, \sigma) = \prod_a p(d_a|\mu, \sigma)$$

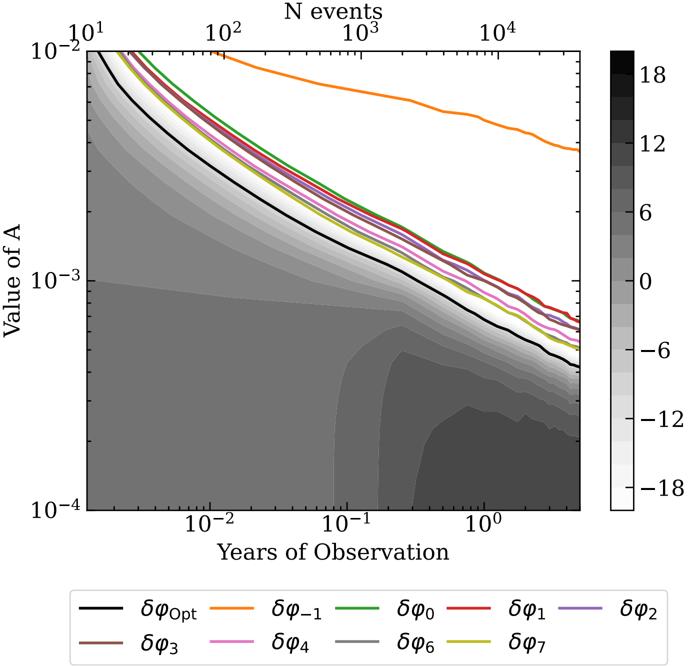
Bayes Factor

- To compare the consistency with GR, we use the log Bayes factor

$$\mathcal{B}_{\mathrm{GR}}^{\mathrm{NVNL}} = \log\left(\frac{p(\{d\}\,|M_{\mathrm{NVNL}})}{p(\{d\}\,|M_{\mathrm{GR}})}\right)$$

  where M_{GR} and M_{NVNL} are the models.
- This is a scalar statistic which quantifies whether GR is preferred (positive) or NVNL (negative).


N events
10^1
10^2
10^3
10^{-1}
10^{-2}
10^{-3}
Value of A
10^{-1}
10^0
Years of Observation
18
12
6
0
−6
−12
−18
$\delta\varphi_{\mathrm{Opt}}$
$\delta\varphi_{-1}$
$\delta\varphi_0$
$\delta\varphi_1$
$\delta\varphi_2$
$\delta\varphi_3$
$\delta\varphi_4$
$\delta\varphi_6$
$\delta\varphi_7$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$\delta\varphi_3$
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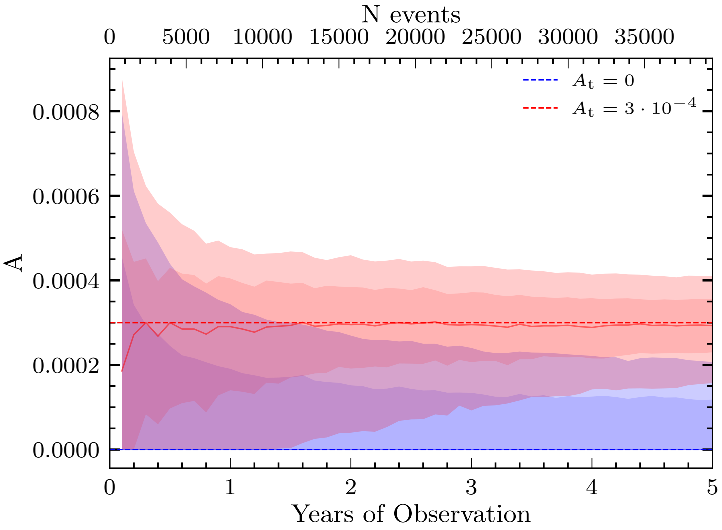
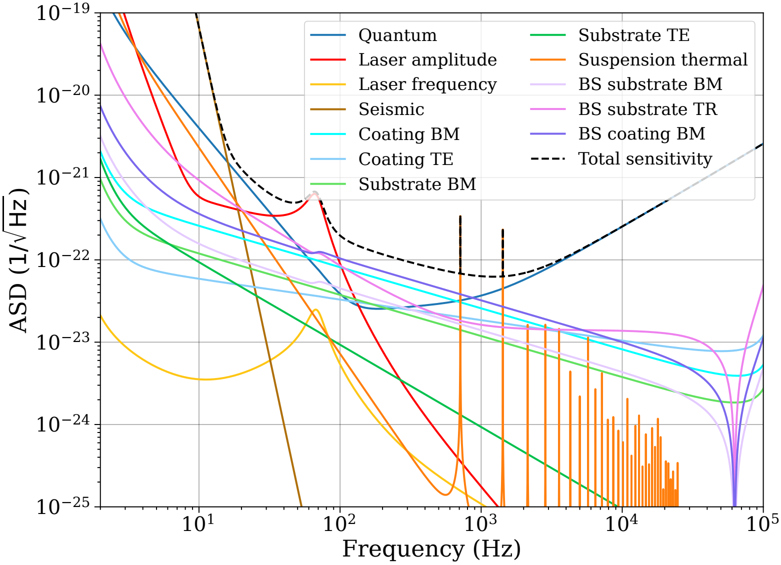
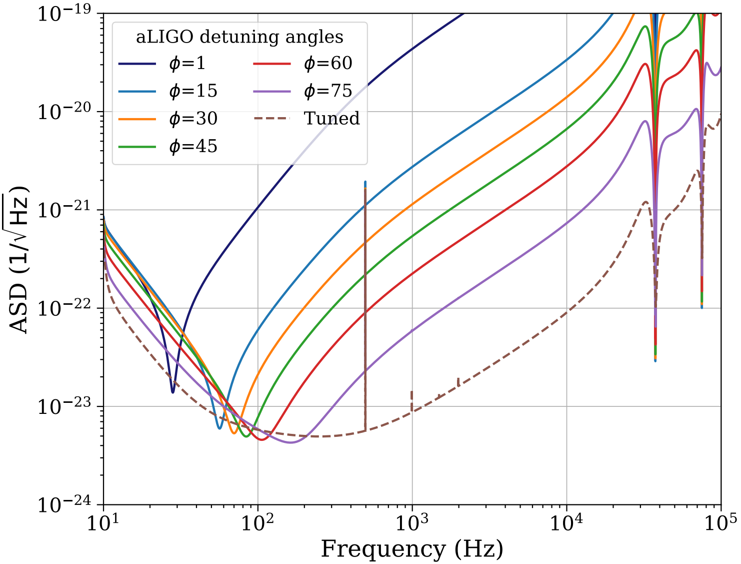
Constraints on Nonviolent Nonlocality with CE



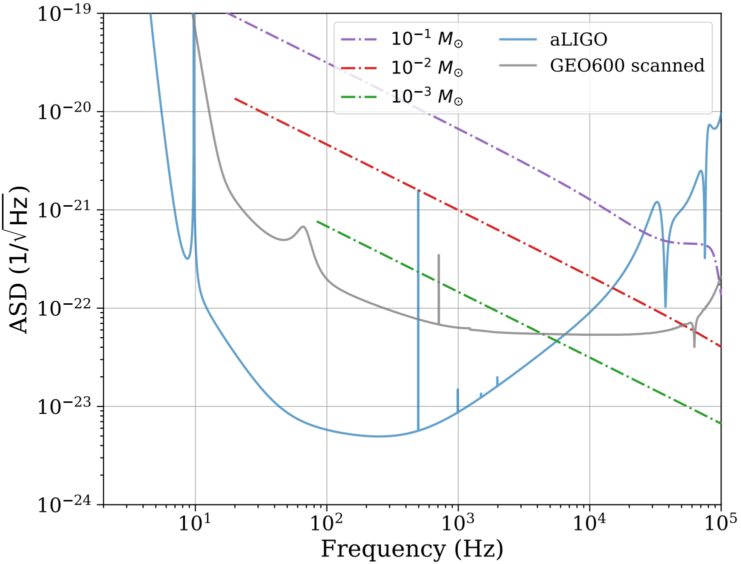
GEO600 Noise Budget



Detuning LIGO



Sub Solar Mass Detuned GEO600



Boson Clouds Full Plot