Testing General Relativity with Gravitational Waves

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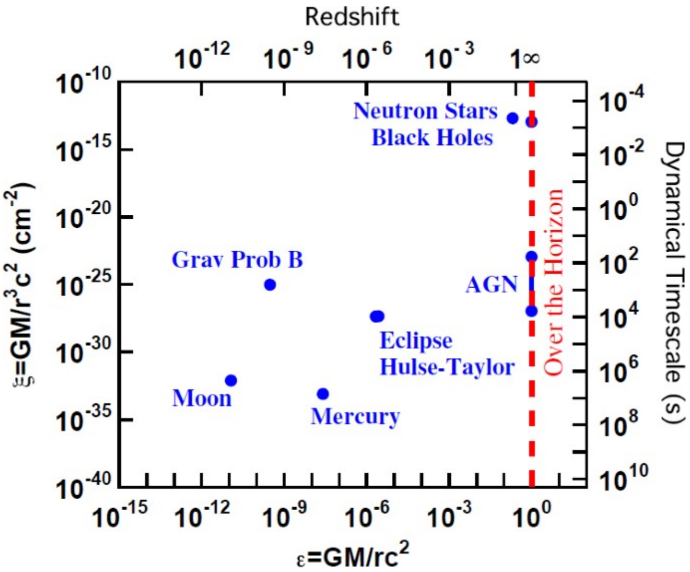
Outline

Landscape of Testing GR



Classical Tests of General Relativity

General Relativity vs Newtonian Gravity

- Are we accelerating right now?

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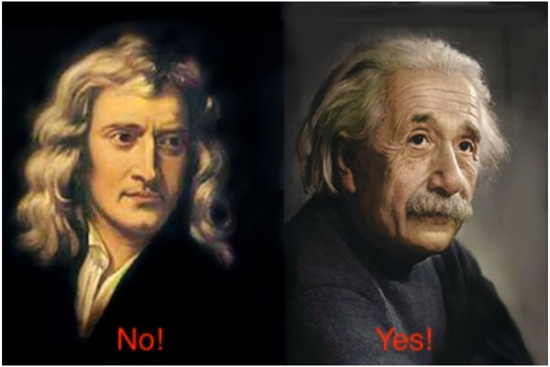

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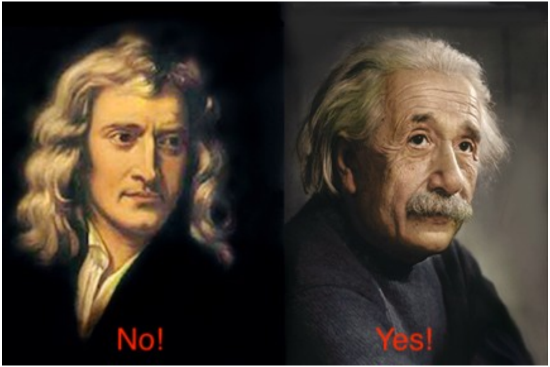
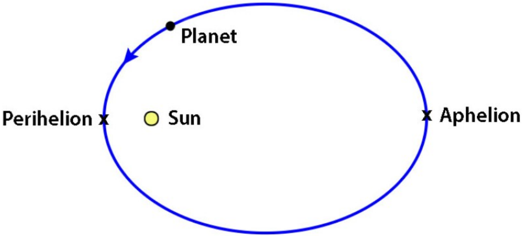


- The floor is exerting a force, and we are accelerating upward. No freefall for us luckily!

Why Are Orbits in Newtonian Gravity Special?



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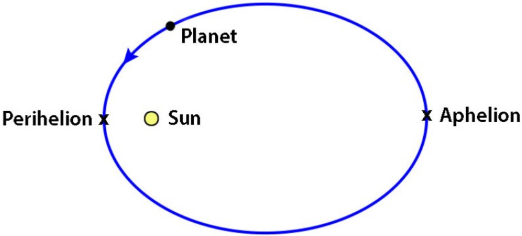

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- Generally for the central force problem, there are six degrees of freedom (x, y, z) and (p_x, p_y, p_z) but only four conserved charges E and $\vec{L}$.

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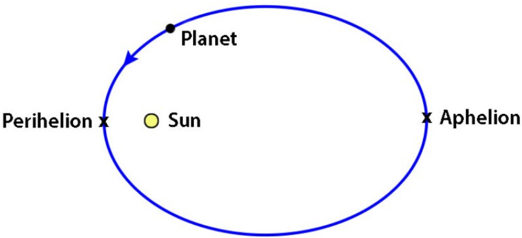
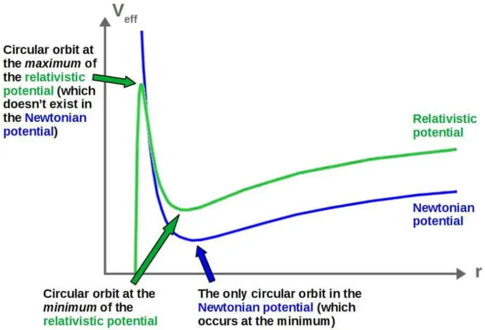


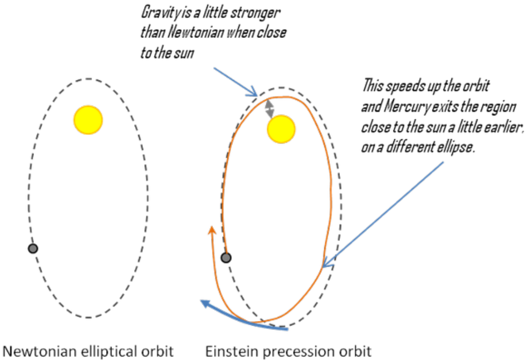
- Kepler found that bound orbits follow and ellipse.
- Generally for the central force problem, there are six degrees of freedom (x, y, z) and (p_x, p_y, p_z) but only four conserved charges E and $\vec{L}$.
- It turns out that there is an additional conserved quantity called the Laplace–Runge–Lenz vector for $V(r) \sim r^{-1}$ and $V(r) \sim r^2$.

General Relativity vs Newtonian Potential



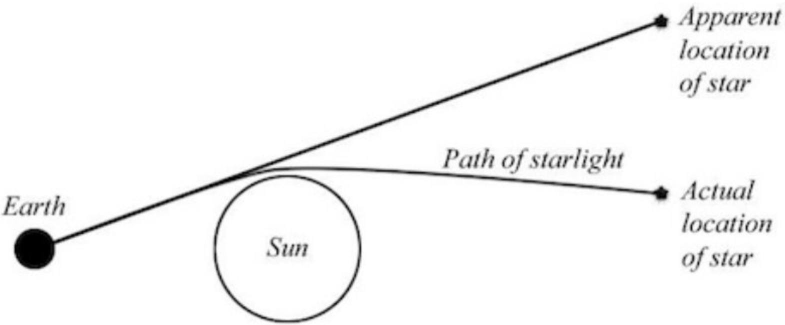
- When you are very close to a start, general relativity deviates from Newtonian gravity.

General Relativity Precession



- Due to the shift in the general relativity potential, the orbit of close bodies such a Mercury will precess
- This was one of the things which proved GR correct.

Deflection of Light



Theory of Deflection of Light

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$$ds^2 = -(1 - 2\phi)\,dt^2 + dx^2 + dy^2 + dz^2 \tag{1}$$

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- If you measure the deflection of light around a massive body, you find that general relativity deflects it twice as much as Newtonian gravity.
- For a nonrelativistic source the Newtonian metric

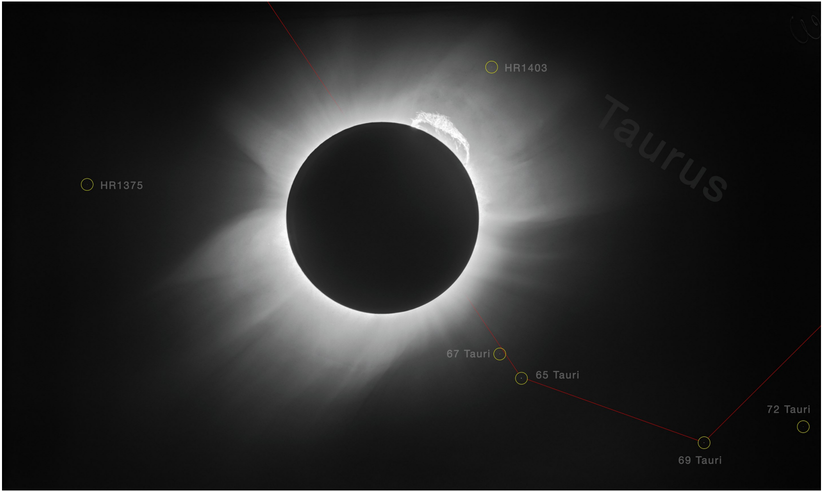
$$ds^2 = -\left(1 - 2\phi\right) dt^2 + dx^2 + dy^2 + dz^2 \tag{1}$$

- However, for a relativistic source, in a weak gravitational field it is

$$ds^2 = -\left(1 - 2\phi\right) dt^2 + \left(1 + 2\phi\right)\left(dx^2 + dy^2 + dz^2\right) \tag{2}$$

- Thus objects traveling quickly with velocity $v \sim c$ [such as light] will deflect very differently!

Eddington experiment 1919



Gravitational Redshift of Light

- Due to the equivalence principle, the light emitted will be Doppler shifted as it enters a gravitational potential.

$$1 + z = \frac{\lambda_\infty}{\lambda_\mathrm{e}} = \left(1 - \frac{r_\mathrm{S}}{R_\mathrm{e}}\right)^{-\frac{1}{2}} \tag{3}$$

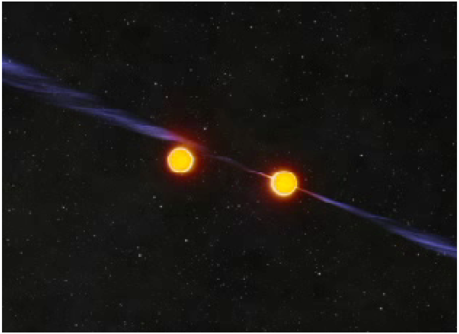
- For earth, the redshift is extremely small around 10^{-8}.

Interstellar Gargantua BH

- How did the astronauts survive the blueshift from the supermassive BH in interstellar? If time dilation is 1 hour per 7 years, then light has its frequency multiplied by 60000!
- Maybe the waves aren't the biggest problem.

Modern Tests of General Relativity

Binary Pulsar : Indirect Evidence of Gravitational Waves



- A pulsar is a rapidly neutron star with a strong magnetic field.
- By observing the radio pulses, you can measure the masses of a binary, and their dynamics.
- Above is an illustration of a white dwarf - pulsar system that was used by Hulse and Taylor to find indirect evidence of GW.

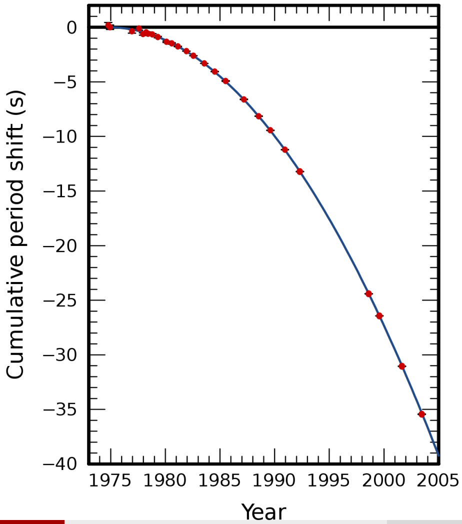
Measuring Pulses

- By seeing how a pulsar's pulses line up, we can measure binary parameters with various effects.
- Below is the first pulses measured in 1967 with the discovery of a pulsar [1].

[1] The image is actually an album that my friend Josh has a shirt of.

Orbital Decay Rate of Hulse Taylor Pulsar

Nobel Prize in 1993 – indirect detection of gravitational waves!

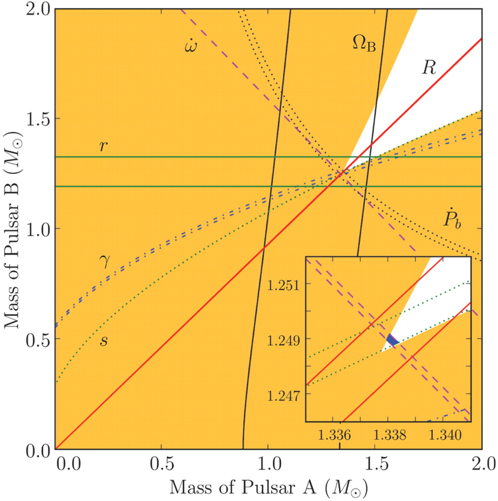


Double Pulsar

- There are many pulsar binaries with white dwarf companions or neutron star companions. However there is only one binary with both objects being a detectible pulsar[2].
- Having both pulses available for measurement is incredibly useful for testing GR, this system gives the best constraints of orbital parameters.

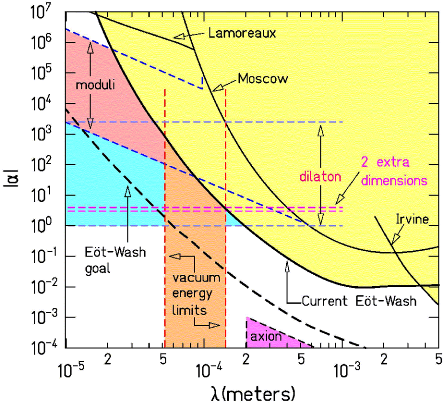
[2] As far as I know, it is an open question whether all neutron stars are pulsars or not.

Double Pulsar

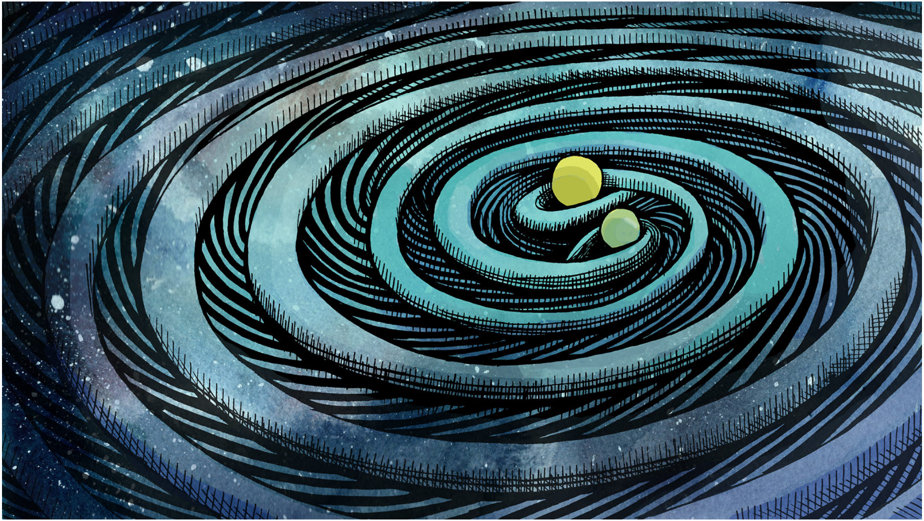


Inverse Square Law Test

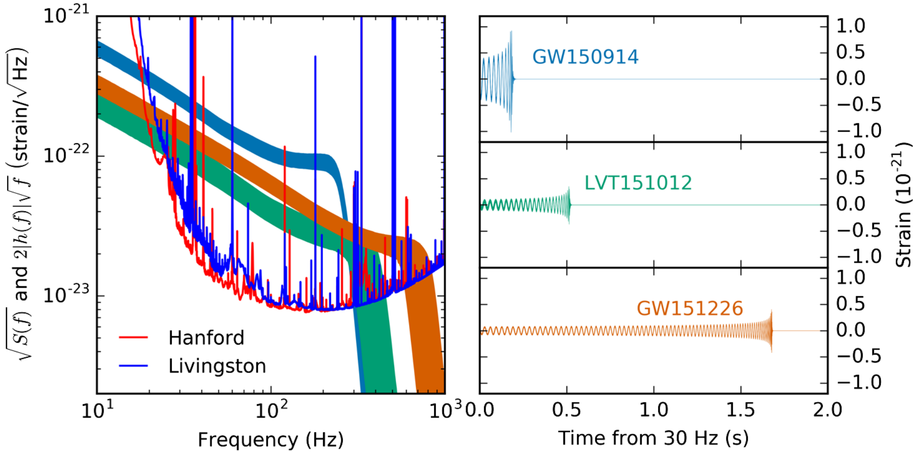
- String theory predicted that there are extra dimensions.
- Table top tests search for these with signatures by looking for deviations of the inverse square law.



Tests of General Relativity with LIGO



Review Gravitational Waves in Freuqency Domain



Modeling Gravitational Waves in GR

- We want to describe them in the frequency domain.

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$$\Psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128\eta}(\pi M f)^{-5/3} \tag{5}$$

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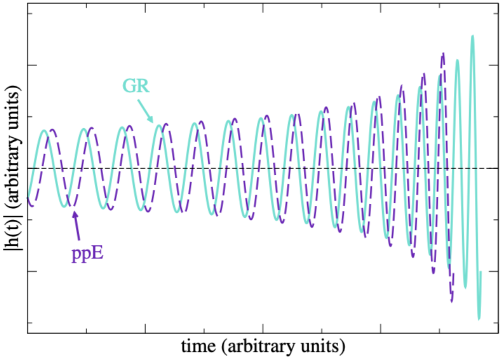
- Λ is due to the antenna pattern of the GW instrument

Effect of Dephasing in Time Domain

- Goal: We want to rewrite the inspiral in a way that we can describe beyond GR gravitational waves like the following:



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[3] Carson et al 2011.02938

Parameterized Deviations from GR

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$$\Psi(f) = 2\pi f t_c - \phi_c - \frac{\pi}{4} + \frac{3}{128\eta}(\pi M f)^{-5/3} \sum_{i=0}^{7} \phi_i (\pi M f)^{i/3} \tag{7}$$

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- If you look at the LIGO papers, they bound the deformation parameters $\delta\phi$. They are the fractional deviation to the above term.

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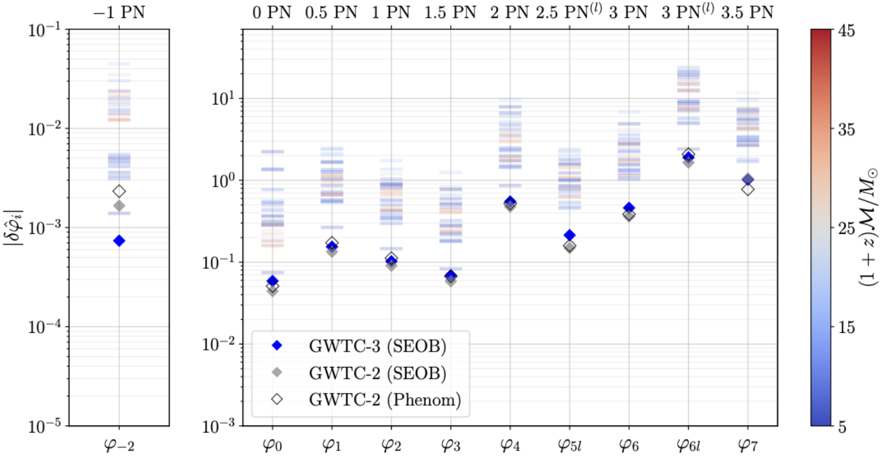
- Note that I may show some plots with β which is proportional to the deformation parameters[4]

$$\beta \propto \delta\phi_n \tag{9}$$

[4] [illegible]

Results for Constraints on Deformation Parameters

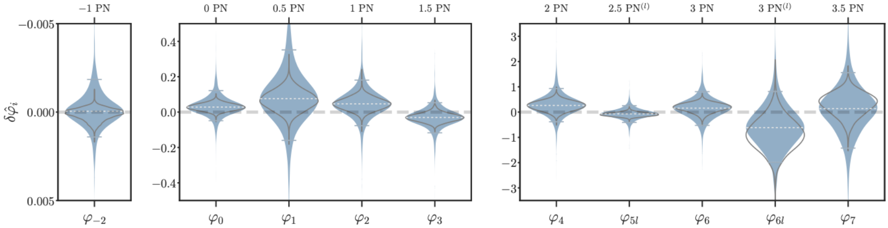
Below is constraints for each event [each line][5]



[5] LIGO GWTC3 2112.06861

Results for Constraints on Deformation Parameters

Combining constraints for each event we get the following[6]



6 LIGO GWTC3 2112.06861

How do We Interperate Parameterized Constraints

- The frequency domain phase is intimately related to the how the frequency of the binary chirps [click to hear chirp].

[7] application to beyond GR in Yunes et al 0909.3328, Tahura et al 1907.10059

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- The frequency domain phase is intimately related to the how the frequency of the binary chirps [click to hear chirp].
- The frequency chirp is related to the energy loss rate in the system by

$$\frac{df}{dt} = \frac{df}{dE}\frac{dE}{dt} \tag{10}$$

- $\frac{df}{dE}$ is due to modified Kepler's third law, $\frac{dE}{dt}$ due to extra energy loss [eg radiation].

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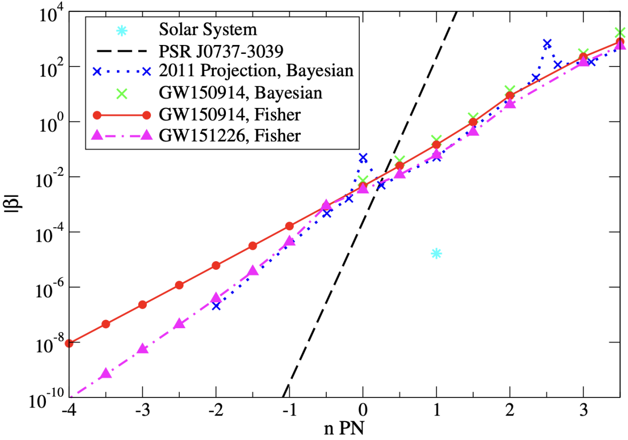
- $\frac{df}{dE}$ is due to modified Kepler's third law, $\frac{dE}{dt}$ due to extra energy loss [eg radiation].
- Solving the above equation for $t(f)$ and then using the following relation which is true from the stationary phase approximation for an adiabatic energy loss rate [7]

$$t(f) = \frac{1}{2\pi}\frac{d\Psi(f)}{df} \implies \Psi(f) = 2\pi \int df\, t(f) \tag{11}$$

[7] application to beyond GR in Yunes et al 0909.3328, Tahura et al 1907.10059

How do GW and Binary Pulsars Compare



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8 Yunes et al 1603.08955

Electromagnetism Example

- As an example, I am going to show how one could bound whether black holes have electric charge. Let us say that each BH has an electric charge which is

$$q = \epsilon \frac{m}{\sqrt{k}} \tag{12}$$

- The binding energy is

$$E_{\mathrm{em}} = k \frac{q_1 q_2}{r} = \epsilon^2 \frac{m_1 m_2}{r} \tag{13}$$

- The energy loss rate for a circular orbit (Lamor formula)

$$\dot{E}_{\mathrm{em}} = \sum_i k \frac{2 q_i^2 a_i^2}{3} \approx \epsilon^2 \frac{4 m_1^2 m_2^2}{3 r^4} \tag{14}$$

Conservative Modification to FD Phase

You can show that

$$\frac{d(E_{\mathsf{N}} + E_{\mathsf{em}})}{df} = \frac{dE_{\mathsf{N}}}{df}\left(1 + \frac{2}{3}\epsilon^2\right) \tag{15}$$

We find $\dot{f} = \frac{df}{dE}\frac{dE}{dt}$

$$\frac{df}{dt} = \left.\frac{df}{dt}\right|_{\mathsf{gr}}\left(1 + \frac{2\epsilon^2}{3}\right) \tag{16}$$

We can end up showing that

$$\Psi(f) = \Psi_{\mathsf{gr}}(f)\left(1 + \frac{2\epsilon^2}{3}(\pi M f)^{0/3}\right) \tag{17}$$

Thus,

$$\frac{2\epsilon^2}{3} \sim \delta\varphi_0 \sim 10^{-1} \implies \epsilon \lesssim 0.4 \implies q \lesssim 2 \times 10^{21} \text{ Coulombs} \tag{18}$$

Dissipative Modification to FD Phase

The energy loss rate can be found to be

$$\dot{E} = \dot{E}_{\mathrm{gr}} \left(1 - \epsilon^2 \frac{5\pi^{2/3}}{72} (Mf)^{-2/3} \right) \tag{19}$$

$$\dot{f} = \dot{f}_{\mathrm{gr}} \left(1 - \epsilon^2 \frac{5\pi^{2/3}}{72} (Mf)^{-2/3} \right) \tag{20}$$

Thus the phase is

$$\Psi(f) = \Psi_{\mathrm{gr}}(f) \left(1 + \epsilon^2 \frac{5\pi^{4/3}}{126} (\pi Mf)^{-2/3} \right) \tag{21}$$

Therefore,

$$\epsilon^2 \frac{5\pi^{4/3}}{126} \sim \delta\varphi_{-2} \sim 10^{-3} \implies \epsilon \leq 0.07 \implies q \lesssim 4 \times 10^{20} \text{ Coulombs} \tag{22}$$

EM Example Conclusion

- Here we showed that we can bound the charge of our black holes.
- The waveform is both modified by (a) changes to the binding energy $E(f)$ and (b) energy dissipation $\dot{E}(f)$.
- Since binding energy deviation occurs as f^0 while the energy loss happens at f^{-2}, the dominant constraint is coming from the latter
- Binding energy modifies the Kepler's 3rd law while this example has scalar dipole radiation.

Massive Gravity

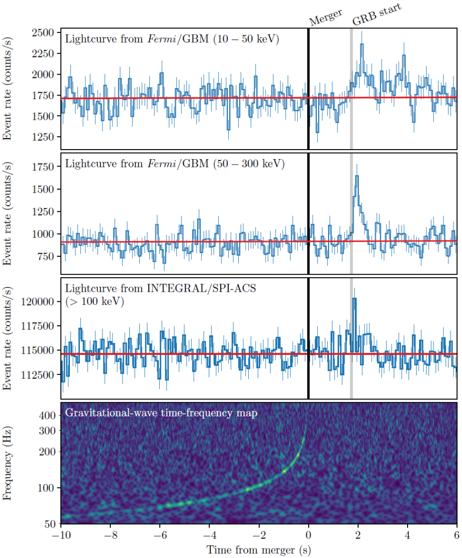
- One interesting thing about gravitation is that the only massless spin 2 field must couple to the stress energy tensor, and thus is GR. This makes it difficult to generalize beyond GR[9].
- One such way is to make the graviton have mass. This would cause it to behave like a Yukawa potential

$$V(r) \propto \frac{e^{-\frac{m_g}{\hbar} r}}{r} \tag{23}$$

- This is motivated by a couple of directions:
    1. Alternatives to Dark Energy for acceleration of expansion of the Universe.
    2. Modification of the galaxy rotation curves instead of dark matter (cf Modified Newtonian dynamics).

[9] Weinberg QFT

Testing GR with GW170817



Constraining the Mass of the Graviton

- If we know that GW170817 had the light delayed by only 2 seconds, how can we bound the graviton mass?
- We know from special relativity

$$E^2 = p^2 + m_g^2 \tag{24}$$

Thus

$$\left(\frac{d\omega}{dk}\right)^2 = v_g^2 = 1 - \frac{m_g^2}{E^2} \tag{25}$$

$$\Delta t = D_L \left(1 - v_g\right) \tag{26}$$

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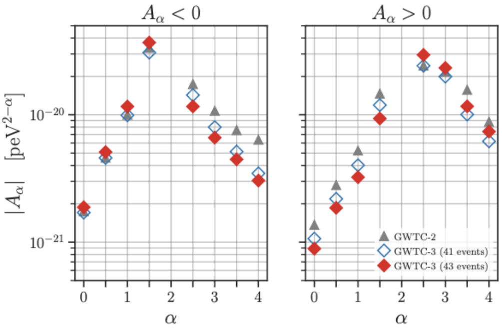
- If $D_L/c = 40$ Mpc$/3 \times 10^8$m/s, and $\Delta t = 2$ sec, and $E = \hbar * 100$Hz then what is m_g?
- You find $v_g/c = 1 - 4 \times 10^{-16}$ and $m_g = 3 \times 10^{-40}$J $= 2 \times 10^{-21}$ eV. This estimation is pretty close to the GWTC3 number of $m_g \leq 1.2 \times 10^{-23}$eV/c^2.

Generalized Dispersion Relation

You can generalize the dispersion to have powers[10]

$$E^2 = p^2c^2 + A_\alpha p^\alpha c^\alpha \tag{27}$$



[10] LIGO GWTC3 2112.06861

Residuals Test

- In any detection, you are measuring a combination of noise and signal.

$$d = n + h \tag{28}$$

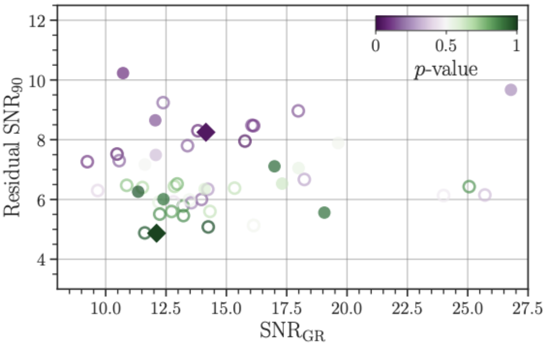
for Gaussian noise, the probability of seeing a particular realization of it is given by the Whittle likelihood

$$p(n) \propto \exp\left[-\frac{1}{2}(n|n)\right] = \exp\left[-\frac{1}{2}\int df \frac{|n(f)|^2}{S(f)}\right] \tag{29}$$

- If you assume that the detector has Gaussian noise, you could see if the residuals have any additional power in them.

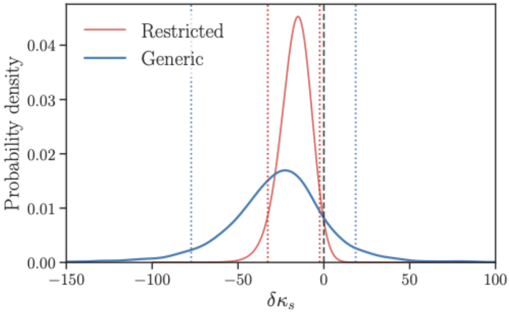
Residuals Test

- The real analysis is much more complicated due to detectors not being perfectly modeled by Gaussians. It is done with Bayeswave, Sophie will know more about this!



Spin Induced Quadrupole

- If black holes are not Kerr, then their quadrupole moment would be different.
- For Kerr BH in GR, the $k_s = 1$.



Ringdown Tests

- Ringdown should be described by

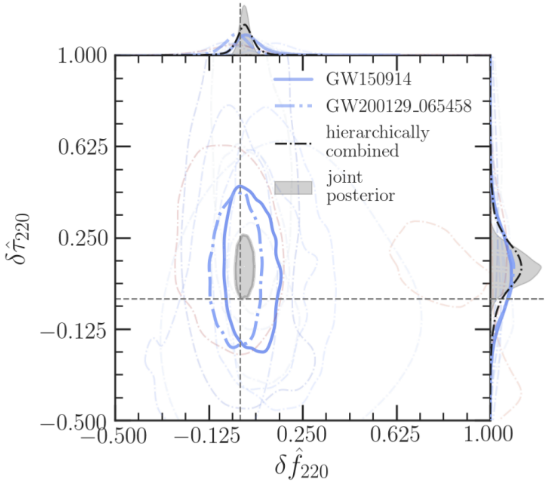
$$h_+(t) - ih_\times(t) = \sum_{\ell=2}^{+\infty} \sum_{m=-\ell}^{\ell} \sum_{n=0}^{+\infty} \mathcal{A}_{\ell mn} \exp\left[-\frac{t-t_0}{(1+z)\tau_{\ell mn}}\right] \exp\left[-\frac{2\pi i f_{\ell mn}(t-t_0)}{1+z}\right] {}_{-2}S_{\ell mn}(\theta, \phi, \chi_\mathrm{f}) \tag{30}$$

- pSEOBNRv4HM ringdown analysis

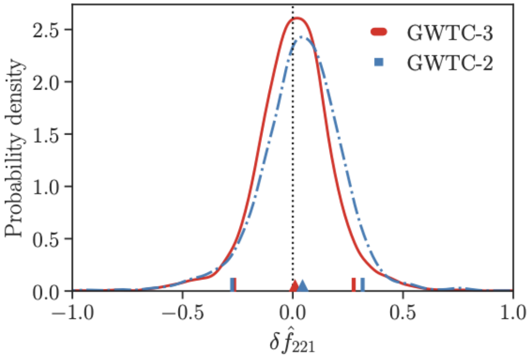
$$\begin{aligned} f_{\ell m0}^{\mathrm{GR}} &= f_{\ell m0}^{\mathrm{GR}}(m_1, m_2, \chi_1, \chi_2), \\ \tau_{\ell m0}^{\mathrm{GR}} &= \tau_{\ell m0}^{\mathrm{GR}}(m_1, m_2, \chi_1, \chi_2). \\ f_{\ell m0} &= f_{\ell m0}^{\mathrm{GR}}\left(1 + \delta\hat{f}_{\ell m0}\right), \\ \tau_{\ell m0} &= \tau_{\ell m0}^{\mathrm{GR}}(1 + \delta\hat{\tau}_{\ell m0}). \end{aligned} \tag{31}$$

- Measure masses and spins in inspiral, and measure QNM with ringdown, and compare deviation.
- Alternatively, you can measure two QNM since Kerr only has two parameters M, χ

Ringdown Tests Results

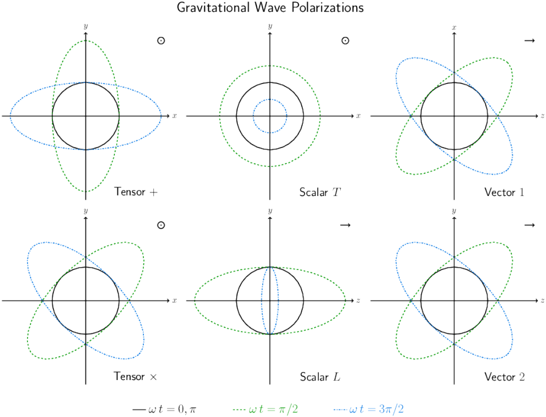


Ringdown Tests Results II



Polarization Test

- If a theory is not general relativity, then there will be additional polarizations of gravitational waves.



Results from GWTC3

TABLE VIII. The table summarizes the choices of basis used in the polarization test. x, $\times$, b, l, x, and y represent the plus mode, cross mode, scalar breathing mode, scalar longitudinal mode, vector x mode, and vector y mode respectively. The first column shows the polarization hypothesis being tested, the third column reports the number of basis modes, and the last column reports the number of free parameters that are marginalized over in the computation of the evidence.

Hypothesis	Description	# of basis modes	Mode(s)	Basis mode(s)	Free parameters
$\mathcal{H}_{T,1}$	Pure tensorial	1	$+, \times$	$+$	5
$\mathcal{H}_{V,1}$	Pure vectorial	1	x, y	x	5
$\mathcal{H}_{S,1}$	Pure scalar	1	b	b	2
$\mathcal{H}_{TS,1}$	Tensor–scalar	1	$+, \times, b, l$	$+$	9
$\mathcal{H}_{TV,1}$	Tensor–vector	1	$+, \times, x, y$	$+$	9
$\mathcal{H}_{VS,1}$	Vector–scalar	1	x, y, b, l	x	9
$\mathcal{H}_{TVS,1}$	Tensor–vector–scalar	1	$+, \times, b, l, x, y$	$+$	13
$\mathcal{H}_{T,2}$	Pure tensorial	2	$+, \times$	$+, \times$	2
$\mathcal{H}_{V,2}$	Pure vectorial	2	x, y	x, y	2
$\mathcal{H}_{TS,2}$	Tensor–scalar	2	$+, \times, b, l$	$+, b$	11
$\mathcal{H}_{TV,2}$	Tensor–vector	2	$+, \times, x, y$	$+, x$	11
$\mathcal{H}_{VS,2}$	Vector–scalar	2	x, y, b, l	x, b	11
$\mathcal{H}_{TVS,2}$	Tensor–vector–scalar	2	$+, \times, b, l, x, y$	$+, b$	19

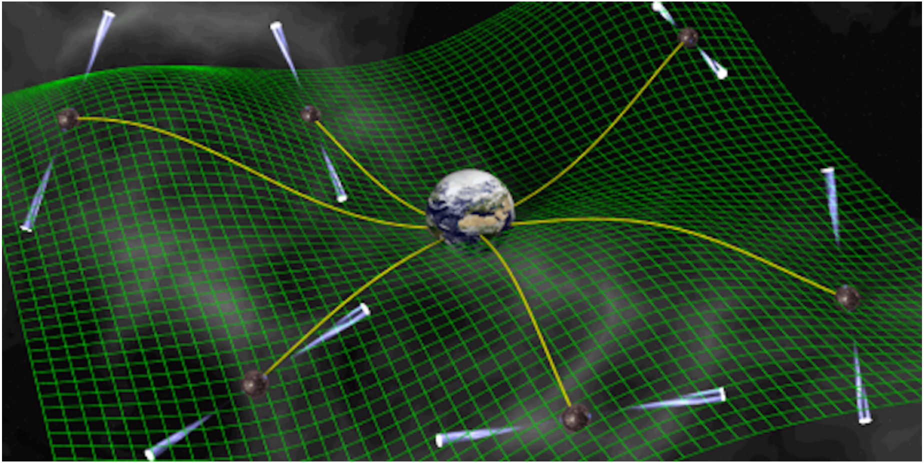
TABLE IX. Combined $\log_{10}$ Bayes factors $\mathcal{B}$ for various polarization hypotheses against the tensor hypothesis, using both 2-detector and 3-detector events. Polarization states have been projected onto one basis-mode as detailed in Sec. VII. Positive (negative) values indicate that the hypothesis indicated in the superscript is favored (disfavored) with respect to the tensorial hypothesis. Error bars refer to 90% credible intervals.

Events	$\log_{10}\mathcal{B}^{S}_{T}$	$\log_{10}\mathcal{B}^{V}_{T}$	$\log_{10}\mathcal{B}^{TS}_{T}$	$\log_{10}\mathcal{B}^{TV}_{T}$	$\log_{10}\mathcal{B}^{VS}_{T}$	$\log_{10}\mathcal{B}^{TVS}_{T}$
O1	-0.04 ± 0.07	0.09 ± 0.07	0.04 ± 0.07	0.09 ± 0.07	0.09 ± 0.07	0.07 ± 0.07
O2	-0.42 ± 0.12	0.04 ± 0.12	0.08 ± 0.12	0.22 ± 0.12	0.09 ± 0.12	0.35 ± 0.12
O3a	-1.85 ± 0.21	-1.04 ± 0.20	0.25 ± 0.20	0.07 ± 0.20	-1.05 ± 0.20	-0.18 ± 0.20
O3b	-1.93 ± 0.17	-0.79 ± 0.17	-0.17 ± 0.17	-0.07 ± 0.17	-0.86 ± 0.17	-0.32 ± 0.17
Combined	-4.24 ± 0.30	-1.70 ± 0.30	0.20 ± 0.30	0.31 ± 0.30	-1.73 ± 0.30	-0.08 ± 0.30

TABLE X. Combined $\log_{10}$ Bayes factor $\mathcal{B}$ for various polarization hypotheses against the tensor hypothesis, for 3-detector events. Polarization states been projected onto two basis-modes as explained in Sec. VII. Positive (negative) values indicate that the hypothesis indicated in the superscript is favored (disfavored) with respect to the tensorial hypothesis. Error bars refer to 90% credible intervals.

Events	$\log_{10}\mathcal{B}^{V}_{T}$	$\log_{10}\mathcal{B}^{TS}_{T}$	$\log_{10}\mathcal{B}^{TV}_{T}$	$\log_{10}\mathcal{B}^{VS}_{T}$	$\log_{10}\mathcal{B}^{TVS}_{T}$
O1	–	–	–	–	–
O2	0.05 ± 0.03	0.01 ± 0.03	-0.02 ± 0.03	0.06 ± 0.03	0.01 ± 0.03
O3a	-0.37 ± 0.12	-0.77 ± 0.12	-0.72 ± 0.12	-0.73 ± 0.12	-0.91 ± 0.12
O3b	-0.09 ± 0.10	-0.22 ± 0.10	-0.35 ± 0.10	-0.38 ± 0.10	-0.38 ± 0.10
Combined	-0.41 ± 0.16	-0.98 ± 0.16	-1.09 ± 0.16	-1.05 ± 0.16	-1.29 ± 0.16

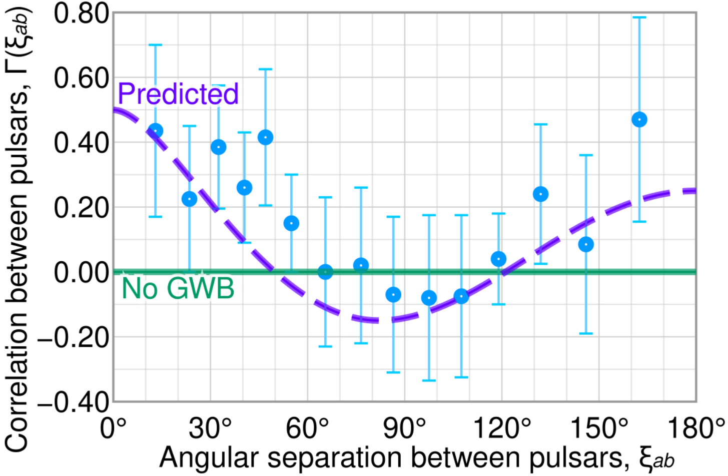
Stochastic Background



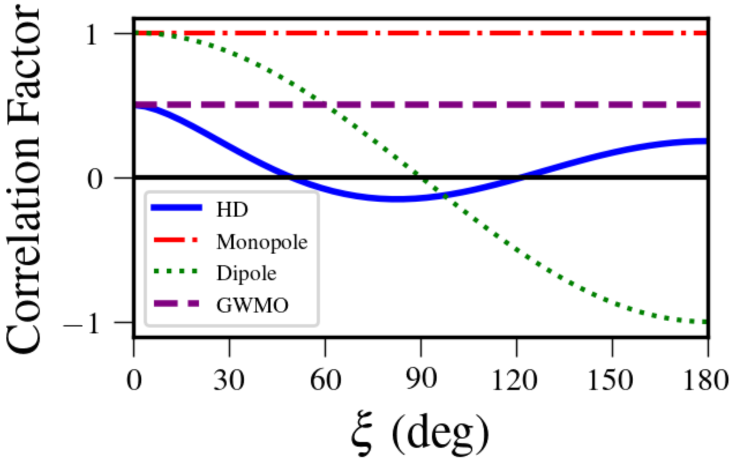
Stochastic Background

- Hellings-Downs curve detected by radio astronomers such as Nanograv:
- Background from many binary black holes. Pat Meyers will probably give a talk which have more detail.
- The stochastic background allows us to measure extra polarizations of GW if we only have less than 5 detectors.

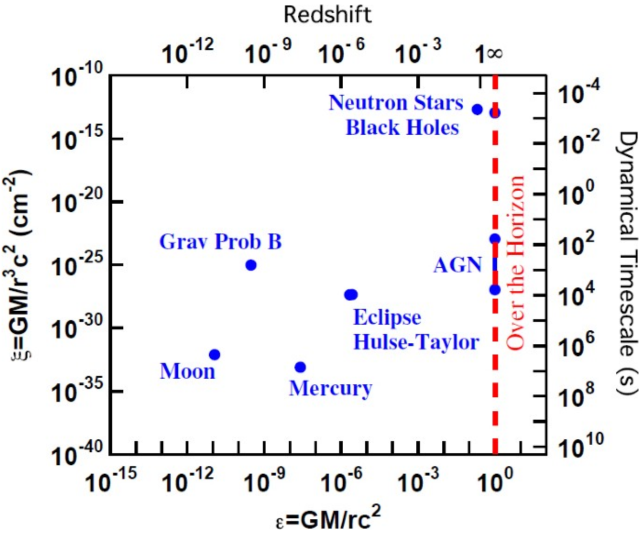
Expected Pattern



Stochastic Background Polarizations Beyond GR



Revisiting Landscape of Testing GR



Conclusion

- Solar system tests are good for incredibly precisely measuring effects, though they are in the weak field and slowly moving regime.
- Pulsars can tell you how gravity behaves for strongly self gravitating objects which are moving slowly.
- Gravitational waves can help test general relativity when curvature is strong, velocities are near the speed of light, and the gravitational field is highly dynamical.

Suggested References for Further Reading

- Michelle Maggiore "Gravitational Waves" I and II ❤️
- Poisson and Will "Gravity: Newtonian, Post-Newtonian, Relativistic"
- Clifford Will "Theory and Experiment in Gravitational Physics"
- Clifford Will 1403.7377 – review paper
- Yunes et al 0909.3328 – parameterized tests