Testing General Relativity with Black Hole-Pulsar Binaries

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Overview

Motivation for Testing General Relativity

- General relativity is currently the most well-tested theory of gravity.
- Nevertheless, it must be an effective field theory of some quantum theory of gravity.
- Gravity has been tested very stringently in the weak field through solar system and cosmological observations.
- It has been tested less however in the strong field regime.

Tests with Pulsar Timing

- Radio observations of pulsar binaries can be used to find their system and orbital properties through pulsar timing.
- Pulsar timing provides precision tests of gravity and has placed stringent bounds on a broad class of theories beyond general relativity.
- Typically this is done with binary pulsar systems such as double pulsar, pulsar-neutron star, and pulsar-white dwarf.

Black Hole-Pulsar Binary

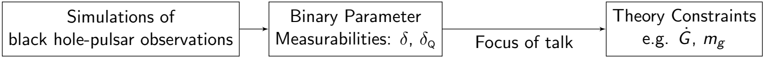
- So far, neither gravitational wave or electromagnetic observations have found a black hole-neutron star binary.
- The Five-hundred-meter Aperture Spherical radio Telescope (FAST) under construction or the next-generation Square Kilometer Array (SKA) may find a binary with a millisecond pulsar orbiting a black hole.
- We will consider the possibility of testing general relativity if a radio telescope finds a black hole-pulsar binary.
- If found, a black hole-pulsar binary would be a powerful test of general relativity.

Measurable Quantities

- Pulsar timing can be used to measure binary parameters such as masses, orbital period, et cetera.
- Specifically, two quantities are of particular importance for this presentation.
  - The orbital decay rate is the time derivative of the orbital period $\dot{P}$.
  - The black hole quadrupole moment Q.
- I will denote the δ and δ_Q to be the fractional error of the orbital decay rate and black hole quadrupole moment respectively.

Methodology

- Measurements of the orbital decay rate and quadrupole moment place constraints on the upper bound of theory parameters.
- Essentially, the maximum possible upper bound on violation from general relativity is constrained by the measurement error.
- Since a black hole binary has not been found yet, we must instead rely on simulated measurement uncertainties to test gravity.



Utility of Black Hole-Pulsar Tests

- Black hole-pulsar binaries are powerful tests of general relativity due to their slower relative velocity (compared to other pulsar binaries).
- The relative velocity of a binary is given by $v = (2\pi M/P)^{1/3}$. Although the mass is larger, the slower orbital period more than compensates for larger total mass.
- The result is a relative velocity smaller that neutron-pulsar binaries by about a factor of 2.
- As I will show later, this makes black hole-pulsar binaries advantageous for constraining theories which have a dependence on velocity to a negative power.

Orbital Decay Rate Bounds

Orbital Decay Rate in General Relativity

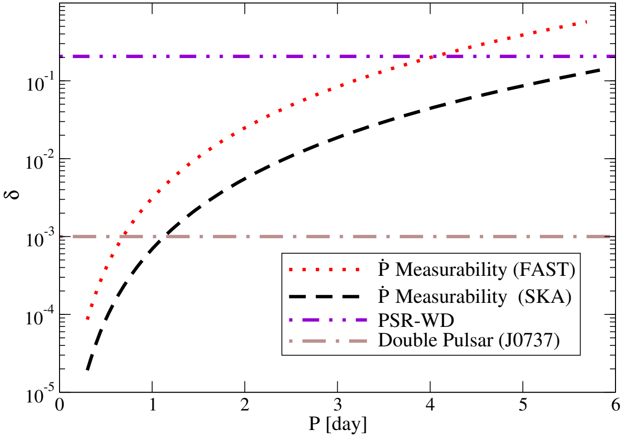
- Orbital decay rate in general relativity is described by the following equation.
- For the rest of this presentation, a subscript with GR represents the quantity in general relativity.

$$\left.\frac{\dot{P}}{P}\right|_{\rm GR} = -\frac{96}{5} G^{5/3} \mu M^{2/3} \left(\frac{P}{2\pi}\right)^{-8/3} F_{\rm GR}(e) \qquad (1)$$

$$F_{\rm GR}(e) \equiv \frac{1}{(1-e^2)^{7/2}} \left(1 + \frac{73}{24} e^2 + \frac{37}{96} e^4\right) \qquad (2)$$

Simulated Orbital Decay Rate Fractional Error

The orbital decay rate fractional error is given by $\left|\frac{\frac{\dot{P}}{P} - \frac{\dot{P}}{P}|_{\mathrm{GR}}}{\frac{\dot{P}}{P}|_{\mathrm{GR}}}\right| < \delta$.



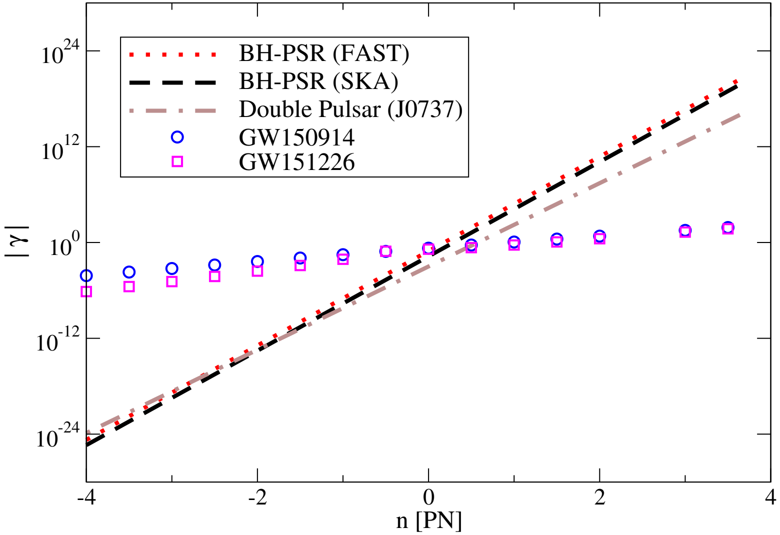
Generic Formalism for Orbital Decay Rate

- We use the following formula characterizing orbital decay rate in modified theories of gravity.

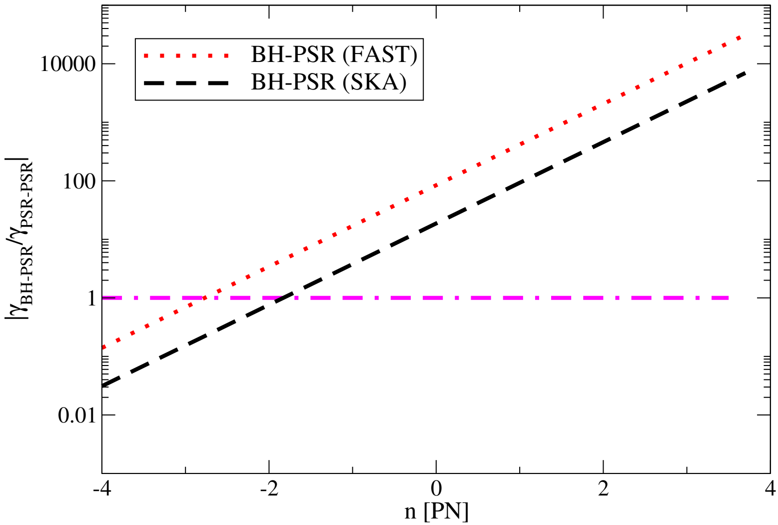
$$\frac{\dot{P}}{P} = \left.\frac{\dot{P}}{P}\right|_{\mathrm{GR}} \left(1 + \gamma\, v^{2n}\right) \tag{3}$$

- The γv^{2n} term gives the leading correction to general relativity where γ is theory dependent and v is the relativity velocity.
- The n gives the post-Newtonian order (PN) of the theory.
- Since the relative velocity of a black hole-pulsar binary is lower than other pulsar binaries, it constrains theories with negative post-Newtonian order more stringently.
- Combining this with the previous slide, we have $|\gamma| < \frac{\delta}{v^{2n}}$.

Astrophysical System Bounds by Post-Newtonian Order



Black Hole-Pulsar and Double Pulsar Comparison

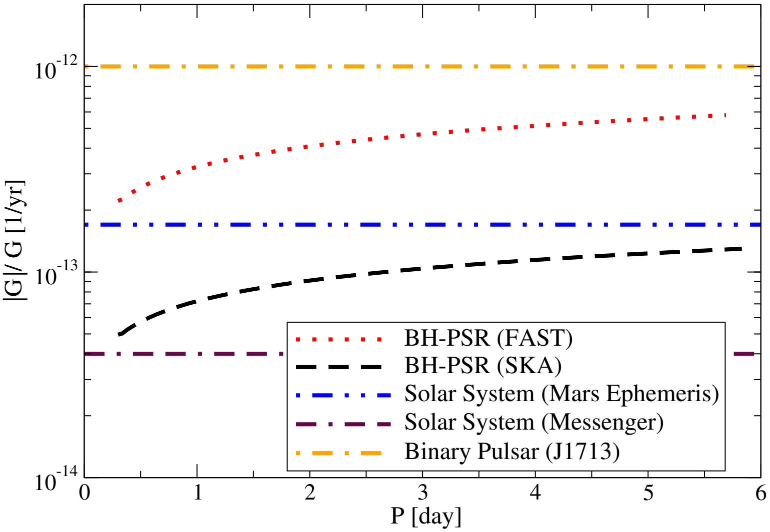


Varying Gravitational Constant

- First, I will consider the constraints possible on a varying gravitational constant.
- The gravitational constants value can be time dependent in many modified theories of gravity.
- For example, the gravitational constant can depend on a scalar field that is coupled to the metric.
- Corrections to orbital decay rate enter in at -4 post-Newtonian order, so a black hole-pulsar binary is very advantageous to constrain this.

$$\frac{|\dot{G}|}{G} < -\frac{1}{2}\frac{\dot{P}}{P}\bigg|_{\mathrm{GR}} \frac{\delta}{1-\left(1+\frac{m_c}{2M}\right)s_p-\left(1+\frac{m_p}{2M}\right)s_c} \tag{4}$$

Varying Gravitational Constant Bounds



Varying Gravitational Constant Discussion

- A black hole-pulsar constraint on $\dot{G}$ is useful to include with stronger solar system measurements for multiple reasons.

1. First, solar system experiments, such as NASA Messenger, measure time variation in G differently than strongly self gravitating bodies (they measure $\partial_t(G\ \mathrm{M}_\odot)/G\ \mathrm{M}_\odot$ instead of $\dot{G}/G$).
2. Binary pulsar measurements capture new effects not present in solar system experiments. This is because there can be a strong field enhancement of the $\dot{G}$ effect in some scalar-tensor theories.

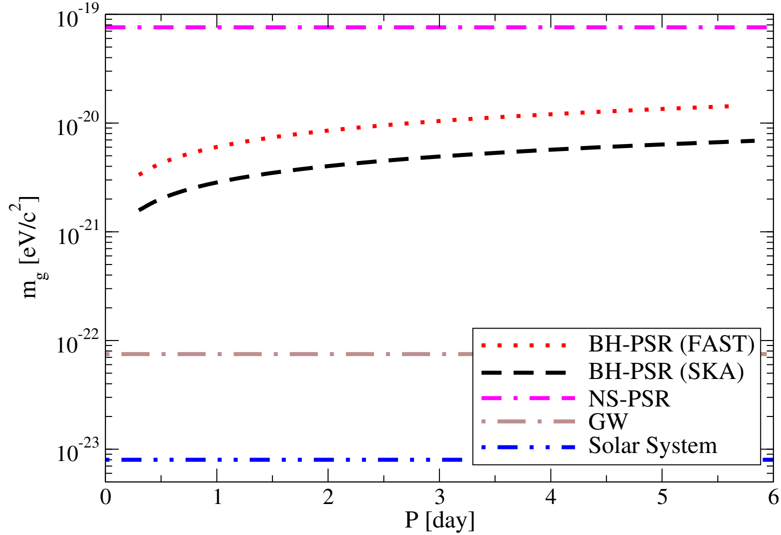
- Thus, black hole-pulsar constraints on $\dot{G}$ provide a complementary bound to solar system experiments.

Lorentz-Violating Massive Gravity

- We will consider bounds on m_g in a Lorentz-violating theory of gravity.
- This is a Fierz-Pauli action with a modified mass term with the following properties.
  1. The $m_g \to 0$ limit recovers linearized general relativity.
  2. The wave equations give standard form in the linearized theory: $(\Box - \bar{m}_g^2)h_{\mu\nu} = -16\pi T_{\mu\nu}$.
- The correction to $\dot{P}$ enters at -3 PN order.

$$m_g^2 \leq \frac{24}{5}(1-e^2)^{1/2} F_{\rm GR}(e) \left(\frac{2\pi\hbar}{c^2 P}\right)^2 \delta \quad (5)$$

Lorentz-Violating Massive Gravity Bounds

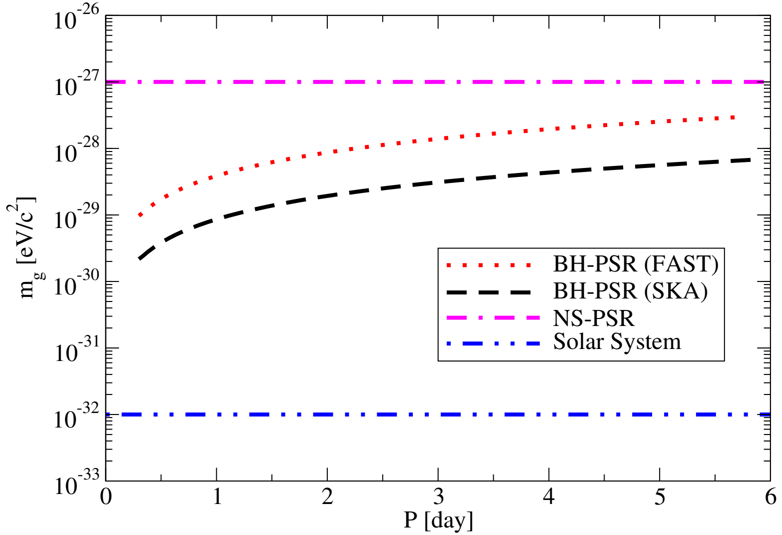


Cubic Galileon Massive Gravity

- We now consider another type of massive gravity capturing the screening effect called the Vainshtein mechanism.
- The Vainshtein mechanism suppresses deviations away from general relativity inside the Vainshtein radius.
- Galileon models are also motivated to explain the accelerating expansion of our universe.
- The largest correction to $\dot{P}$ comes from the quadrupolar radiaion at -2.75 PN.

$$m_g \leq \frac{2^7}{5\lambda^2} \frac{1}{F_{\mathrm{CG}}(e)} \frac{M_{\mathrm{PL}}^3}{M^{\frac{1}{2}} M_{\mathrm{Q}}^2} \frac{1}{\Omega_P^{\frac{1}{2}} (\Omega_P a)^3} L_{\mathrm{GR}}\, \delta \qquad (6)$$

Cubic Galileon Massive Gravity Bounds

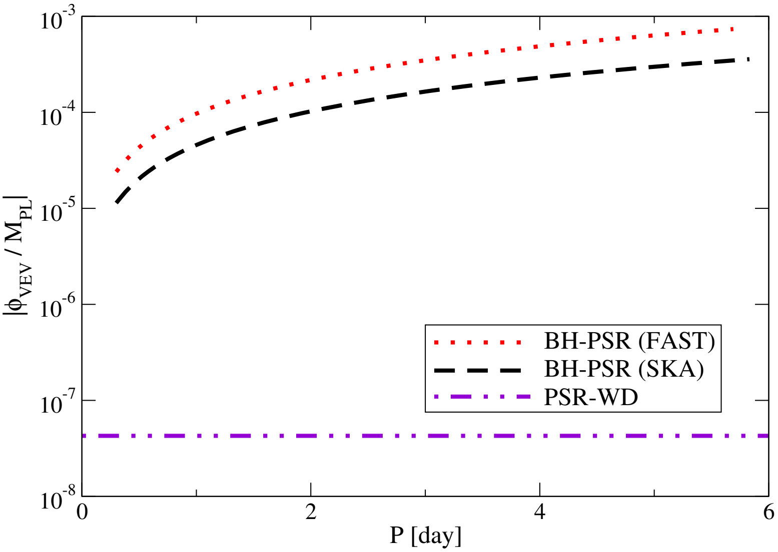


General Screen Modified Gravity

- General screened modified gravity is a scalar modification to GR with a fifth force and screening mechanism.
- The scalar field induces non-GR effects on cosmological scale that can explain current accelerating expansion of our universe without introducing dark energy and induces a screening mechanism in our solar system.
- The correction enters in at the -1PN order.

$$\left|\frac{\phi_{\rm VEV}}{M_{\rm PL}}\right| \leq \frac{m_p}{R_p}\left(\frac{2\pi M}{P}\right)^{1/3}\left[\frac{192}{5}\frac{F_{\rm GR}(e)}{F_{\rm SMG}(e)}\delta\right]^{1/2} \tag{7}$$

General Screen Modified Gravity Bounds



Quadrupole Moment Bounds

Modification of Black Hole Quadrupole Moment

- Previously, we have examined bounds with orbital decay rate. However, if the post-Newtonian order is positive, a black hole-pulsar binary places weaker constraints than other binary pulsar systems.
- In this section, we will examine constraining gravity by measuring the quadrupole moment of either a stellar or supermassive black hole-pulsar binary.
- Any deviation from the Kerr black hole quadrupole moment will modifiy the orbit.

Measurement of Black Hole Quadrupole Moment

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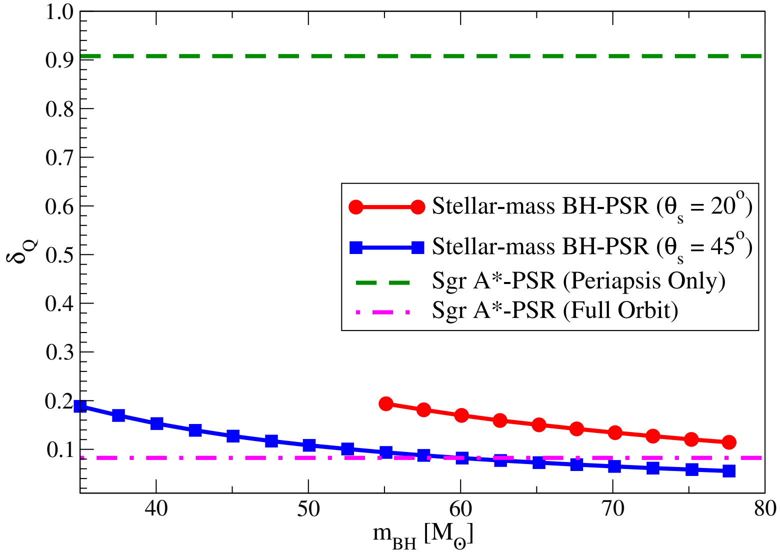
- A non-vanishing quadrupole moment Q of a black hole produces a periodic perturbation of the pulsar's orbit.
- Pulsar timing can measure the quadrupole moment through the Roemer time delay.
- The Roemer time delay is the modulation in travel time for light due to a the pulsar's orbit.
- Specifically, the Roemer time delay is the time for light to travel between the closest and furthest points of a pulsar's orbit to earth.
- The black hole quadrupole moment can be then be extracted by observations of the Roemer delay.

Possible System Binaries

- We will use simulations of the fractional measurement accuracy δ_Q of the black hole quadrupole moment for two cases.
  1. A millisecond pulsar orbiting a stellar-mass black hole.
  2. A millisecond pulsar orbiting Sgr A*.

Simulated Black Hole Quadrupole Moment Measurement Accuracy



Motivation for Quadratic Curvature Modifications

- Recall that the Einstein-Hilbert action in general relativity contains only linear terms in curvature with the Ricci scalar.

$$S_{\mathrm{GR}} = \frac{1}{16\pi} \int d^4x \sqrt{-g} R \tag{8}$$

- In the following sections, we will investigate modifications of general relativity which add scalar fields coupled through quadratic-curvature corrections to the Einstein-Hilbert action.
- This is motivated by various theories of quantum gravity.

Dynamical Chern-Simons

- Dynamical Chern-Simons is a parity-violating and quadratic-curvature theory of gravity with a pseudoscalar field motivated by string theory and loop quantum gravity.
- The pseudoscalar field θ is coupled to the Pontryagian density with coupling constant α_{dCS} in the action $\frac{\alpha_{\mathrm{dCS}}}{4}\theta\, {}^*R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$.
- This modifies the Kerr black hole quadrupole moment (in the small coupling approximation $\zeta_{\mathrm{dCS}} \ll 1$),

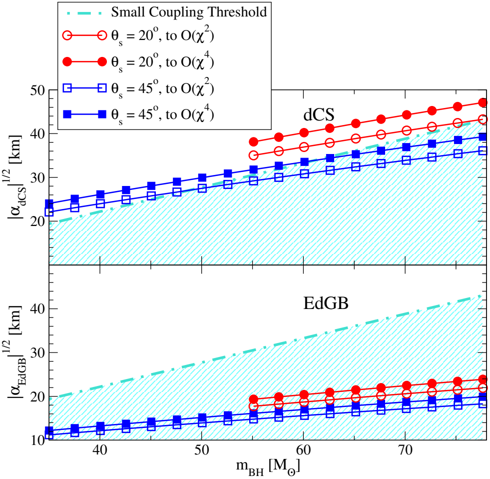
$$Q = Q_{\mathrm{GR,k}} \left(1 - \frac{201}{1792}\zeta_{\mathrm{dCS}} + \frac{1819}{56448}\zeta_{\mathrm{dCS}}\chi^2\right) . \qquad (9)$$

- Thus, constraints can be placed with (using $\zeta_{\mathrm{dCS}} = \frac{\alpha_{\mathrm{dCS}}^2}{\kappa_g m_{\mathrm{BH}}^4}$),

$$\alpha_{\mathrm{dCS}}^{1/2} \leq 4\sqrt{21} \left(\frac{\kappa_g\, \delta_{\mathrm{Q}}}{12663 - 3638\chi^2}\right)^{1/4} m_{\mathrm{BH}} . \qquad (10)$$

Dynamical Chern-Simons Stellar Black Hole Bounds

- Current bounds: $\sqrt{\alpha_{\rm dCS}} = \mathcal{O}(10^8)$ km



Dynamical Chern-Simons Super Massive Black Holes Bounds

- The small coupling threshold for ${\alpha_{\rm dCS}}^{1/2}$ is equal to 3×10^6 km for Sgr A*.
- The strongest possible bound comes from full orbit measurements from a PSR orbiting Sgr A*.
- Unfortunately, such strongest bound is above the small coupling threshold of $\alpha_{\rm dCS}^{1/2}$ by about 20%.
- Thus, our black hole quadrupole formula as no longer valid, so we cannot place bounds on dynamical Chern-Simons with Sgr A*-pulsar measurements.

Einstein-dilaton Gauss-Bonnet

- Einstein-dilaton Gauss-Bonnet is a theory of gravity motivated by string theory with scalar field and a curvature-squared coupling.
- We consider a linear coupling between the scalar field and gravity which adds an extra term $\alpha_{\mathrm{dCS}}\phi R_{GB}^2$ to the action where scalar field ϕ is coupled to the Gauss-Bonnet term with coupling constant α_{EdGB}.
- This modifies the Kerr black hole quadrupole moment (in the small coupling approximation $\zeta_{\mathrm{EdGB}} \ll 1$),

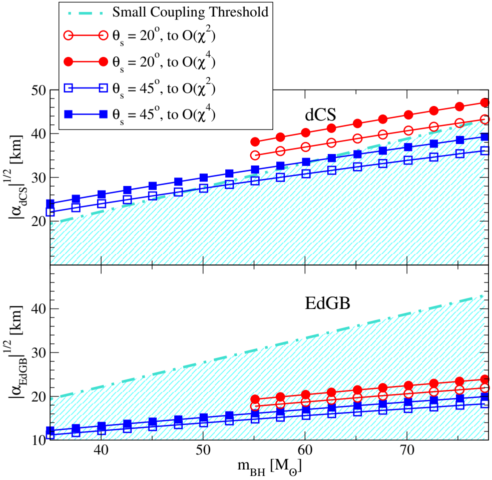
$$Q = Q_{\mathrm{GR,k}} \left(1 + \frac{4463}{2625}\zeta_{\mathrm{EdGB}} - \frac{33863}{68600}\zeta_{\mathrm{EdGB}}\chi^2\right) . \qquad (11)$$

- Constraints can be placed with (using $\zeta_{\mathrm{EdGB}} = \frac{\alpha_{\mathrm{EdGB}}^2}{\kappa_g m_{\mathrm{BH}}^4}$),

$$\alpha_{\mathrm{EdGB}}^{1/2} \leq 3^{1/4}\, 70^{3/4} \left(\frac{\kappa_g \delta_Q}{1749496 - 507945\chi^2}\right)^{1/4} m_{\mathrm{BH}} . \qquad (12)$$

Einstein-dilaton Gauss-Bonnet Stellar Black Hole Bounds

- Current bounds: $\sqrt{\alpha_{\text{EdGB}}} = 1.9$ km



Conclusion

Conclusion

- We have studied how well one can probe alternative theories of gravity with both orbital decay rate and black hole quadrupole moment measurements if a black hole-pulsar binary is found.
- We have shown that a black hole-pulsar binary can place competitive bounds with orbital decay rate modification to theories with negative post-Newtonian order (specifically $\dot{G}$).
- We showed that the Roemer time delay for certain stellar-mass black hole-pulsar configurations can be used to place bounds on dynamical Chern-Simons gravity that are six orders of magnitude stronger than the current most stringent bounds.
- Thus, the detection of a black hole-pulsar binary will allow new tests of gravity.

Future Work

- The black hole quadrupole moment formula in dynamical Chern-Simons could be improved to arbitrary spin through recent numerical developments.
- It is interesting to consider bounds on dynamical Chern-Simons through measurement of advance rate of periastron in a black hole-pulsar binary instead of quadrupole moment measurement.
- This analysis could be extended to black hole-pulsar bounds on Lorentz-violating theories, such as Einstein-æther and khronometric gravity, in combination with new GW170817 constraints.

Acknowledgements and Questions

- I'd like to thank Kent Yagi for his collaboration on this project.
- I'd be happy to take any further questions.
- A summary of orbital decay rate bounds is located on the next slide.

Summary Table for Orbital Decay Rate Modification

Theory	γ	$f(e)$	n	Theoretical parameters	Refs.	Stronger bounds?
Time-Varying G (Sec. II B 1)	$\frac{5\dot{G}M^3}{48m_c m_p}\left[1 - s_p^{\dot{G}}\left(1+\frac{m_c}{M}\right) - s_c^{\dot{G}}\left(1+\frac{m_p}{M}\right)\right] f(e)$	$\frac{1}{F_{\rm GR}(e)}$	-4	$\dot{G}/G$	[17]	✓
Lorentz-violating Massive Gravity (Sec. II B 2)	$\frac{5M^2}{24}m_g^2 f(e)$	$\frac{1}{(1-e^2)^{1/2}F_{\rm GR}(e)}$	-3	m_g	[37]	✗
Cubic Galileon Massive Gravity (Sec. II B 3)	$\frac{25}{32}\pi\lambda^2\frac{M_{\rm PL}M_{\rm Q}^2 M^3}{m_p^2 m_c^2} m_g f(e)$	$\frac{F_{\rm CG}(e)}{F_{\rm GR}(e)}$	$-11/4$	m_g	[38]	✗
General Screen Modified Gravity (Sec. II B 4)	$\frac{5}{192}(\epsilon_p - \epsilon_c)^2 f(e)$	$\frac{F_{\rm SMG}}{F_{\rm GR}(e)}$	-1	$\phi_{\rm VEV}/M_{\rm PL}$	[39]	✗
(massless) Scalar-Tensor	$\frac{5}{96}\left(\bar{\alpha}_p^{\rm ST} - \bar{\alpha}_c^{\rm ST}\right)^2 f(e)$	$\frac{F_{\rm SMG}}{F_{\rm GR}(e)}$	-1	(α_0, β_0)	[19]	✓ [28, 30]
Einstein-dilaton Gauss-Bonnet	$\frac{5\pi}{24}\left(\bar{\alpha}_p^{\rm EdGB} - \bar{\alpha}_c^{\rm EdGB}\right)^2 f(e)$	—	-1	$\sqrt{\alpha_{\rm EdGB}}$	[31]	✓ [31]
Einstein-Æther	$\frac{5C_{\rm EA}}{32}\left(1-\frac{c_{14}}{2}\right)\left(s_p^{\rm EA} - s_c^{\rm EA}\right)^2 f(e)$	—	-1	(c_+, c_-)	[40]	?
Khronometric	$\frac{5C_{\rm kh}}{32}\left(1-\frac{\alpha_{\rm kh}}{2}\right)\left(s_p^{\rm kh} - s_c^{\rm kh}\right)^2 f(e)$	—	-1	$(\lambda_{\rm kh}, \alpha_{\rm kh}, \beta_{\rm kh})$	[40]	?

TABLE I. Mapping between non-GR parameters (γ and n) in the orbital decay rate $\dot{P}$ in Eq. (1) to theoretical parameters in various example modified theories of gravity, together with some references. These expressions are valid for any compact binaries (not specific to BH-PSRs). The first four theories are those considered in Sec. II B, while the last four theories are presented only for reference. Note that we study bounding EdGB gravity in Sec. III B 2 via BH quadrupole moment measurement which is different from the orbital decay rate presented here. Theoretical parameters are presented in the fifth column. The last column shows whether BH-PSR bounds are stronger than other existing bounds (✓: yes; ✗: no; ?: unknown). The meaning of each parameter in the second column is as follows. m_p: primary PSR's mass, m_c companion's mass, M: total system mass, e: eccentricity, $M_{\rm PL}$: Planck mass, $M_{\rm Q}$: a mass parameter in Eq. (21), λ: a numerical constant in Eq. (22), ϵ_A: a screening parameter in SMG in Eq. (29), $C_{\rm EA}$ and $C_{\rm kh}$: a function of theory parameters in Eqs. (114) and (124) of [40] for Einstein-æther and khronometric theories respectively, c_{14} and $\alpha_{\rm kh}$: a combination of coupling constants in Einstein-æther theory and khronometric theory respectively. $\bar{\alpha}_A$ is the scalar charge. In many of scalar-tensor theories, it is non-vanishing for stars while it is zero for a BH [41–43]. In EdGB gravity, such a charge vanishes for stars [31, 44] while that for a BH is in Eq. (37) of [45]. s_A is the sensitivity and that for a neutron star in Einstein-æther and khronometric theory has been computed in [40, 46][a] while that for a BH has not been calculated yet. The eccentricity dependent function $f(e)$ is presented in the third column if known, while "—" means that the correction has been calculated only for circular binaries ($f = 1$). $F_{\rm GR}$ is the eccentricity dependence in GR in Eq. (3), while $F_{\rm CG}(e)$ and $F_{\rm SMG}(e)$ are that in cubic Galileon massive gravity and generic screened massive gravity defined in Eqs. (24) and (28) respectively.

[a] The fitting function for the NS sensitivity in Einstein-æther and khronometric theory can be found in Eq. (186) or (C1) of [40], though the parameter region in which the fit is valid has mostly been ruled out by GW170817 [47, 48].